Properties of the transmission line matrix model for outdoor sound propagation: Numerical dispersion effects

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Contents

Introduction & context

- Acoustic propagation in forest environments

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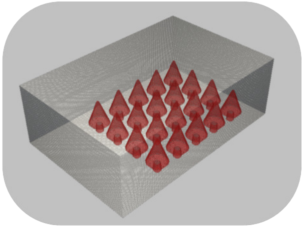
  - Multi-scale
  - Multiple scatterers
  - Impedance boundary conditions



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- Acoustic propagation in forest environments
  - Multi-scale
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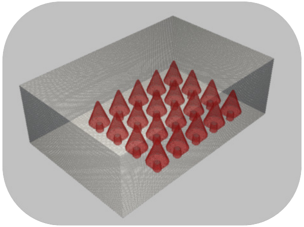
- Various sources



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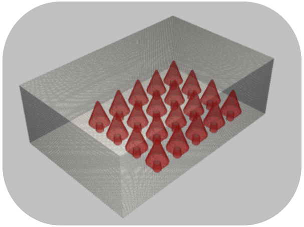
- European directive [2002/49/EC]: protection of quiet areas



Numerical model

History

Transmission Line Matrix model (TLM)

Introduced to model high frequency magnetic fields [Johns and Beurle, 1971].

- Time-domain method

Numerical model

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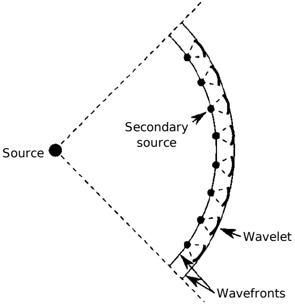
- Python/OpenCL parallelized on GPUs [Guillaume and Fortin, 2014]

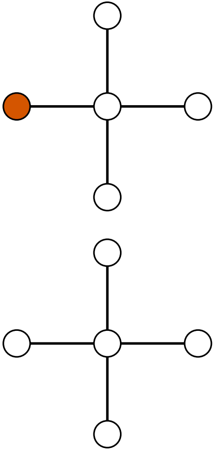
Numerical model

Basis



Huygens principle [Guillaume, 2009].

⊕ detailed : [Goestchel et al., 2022]



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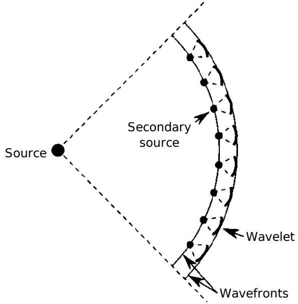

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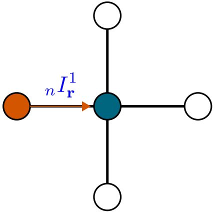

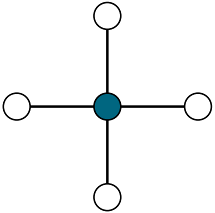

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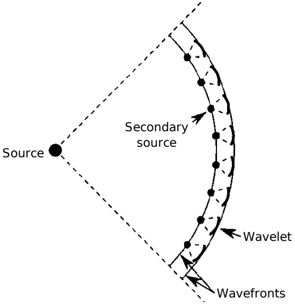

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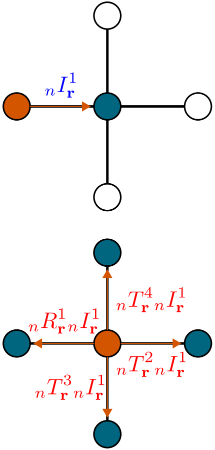


Numerical model

Stability study

Homogeneous case

$$ {}_{n+1}P_{\mathbf{r}} + {}_{n-1}P_{\mathbf{r}} = \frac{1}{d}\sum_{m=1}^{d}\left[{}_{n}P_{(j_1+\delta_{m1},\dots,j_d+\delta_{md})} + {}_{n}P_{(j_1-\delta_{m1},\dots,\, j_d-\delta_{md})}\right], \tag{1} $$

$$ \frac{\partial^2 p}{\partial t^2} - c_{\mathrm{TLM}}^2 \nabla^2 p = \mathcal{O}\left(\Delta t^2\right) + \mathcal{O}\left(\frac{\Delta \ell^4}{\Delta t^2}\right), \text{ with } c_{\mathrm{TLM}} = \frac{\Delta \ell}{\sqrt{d}\Delta t}. \tag{2} $$

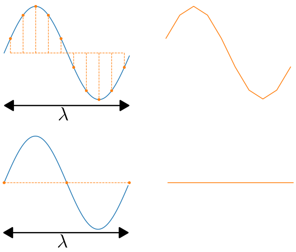
Axial dispersion

$$ \cos(\omega \Delta t) = \frac{1}{d}\sum_{m=1}^{d}\cos(k_{x_m}\Delta \ell) \quad \forall \Delta t,\ \forall \Delta \ell. \tag{3} $$

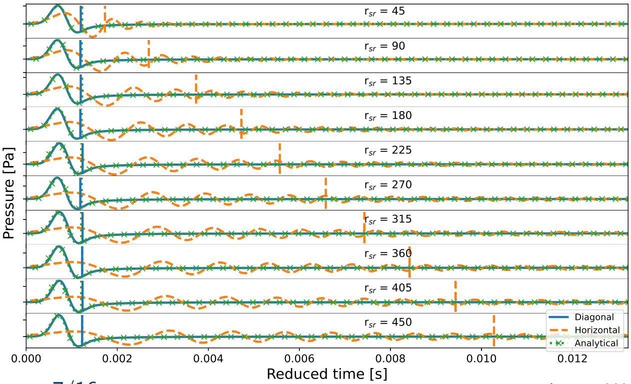
Numerical model

Dispersion effect on results

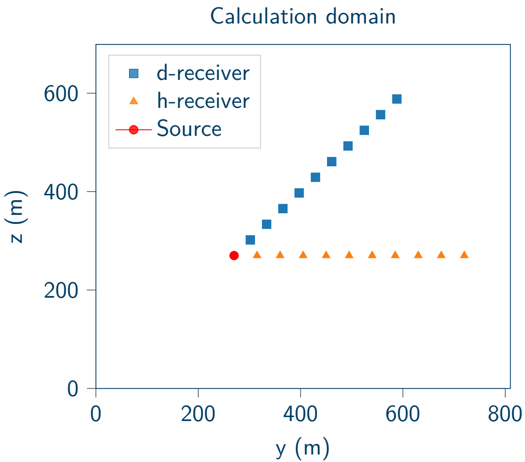
Number of points per wavelength (N_{ppw})



Cartesian mesh numerical dispersion



Free field setup

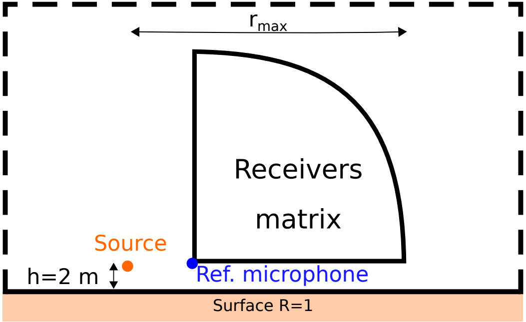


Configuration for $f_{max} = 2400$ Hz, $\Delta\ell = \lambda/10$, $r_{max} = 450$ m:

$\Delta\ell$ [m]	Δt[s]	$\Delta\ell_{mic}$[m]	$N_y \times N_z$ [-]
0.014	2.95E-05	45	56476*48746

Memory [Gb] (float32 numpy array)	t_{sim}[s]	r_{max}/λ_{min} [-]
10.22	11143	3176

Reflective ground setup



Normalisation & results

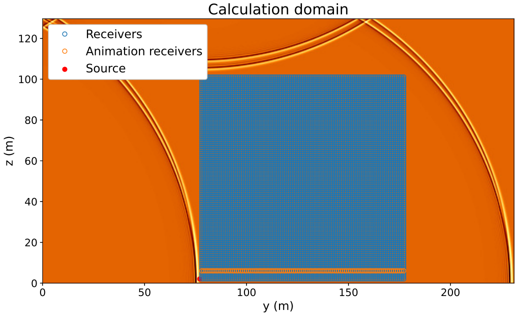
- Normalisation: standard deviation ratio

$$m_{ij}^n = \frac{m_{ij}^n \times \sigma(p_{0j}^n)}{\sigma(m_{0j}^n)}$$

- Attenuation // reference microphone:

$$Att(r) = 10 \times \log \left(\frac{\sum_n p_{ij}^2[n]}{\sum_n p_{0j}^2[n]} \right)$$

Reflective ground setup



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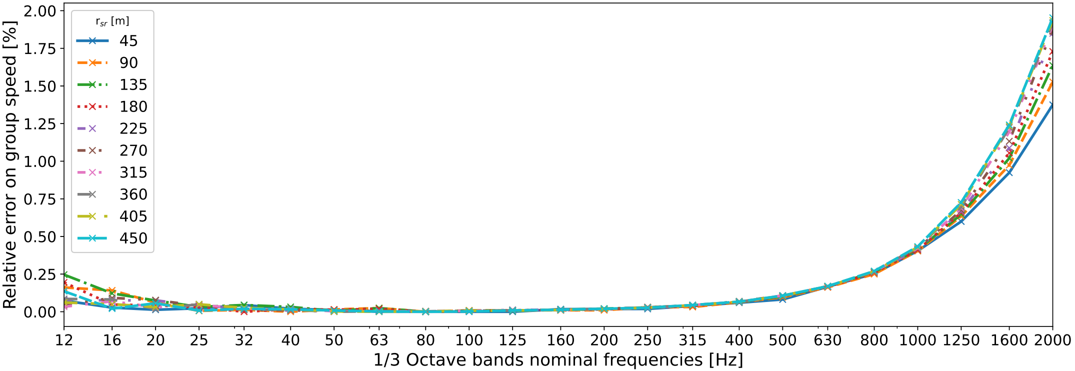
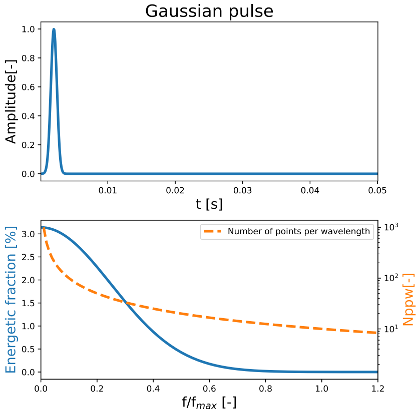
Results

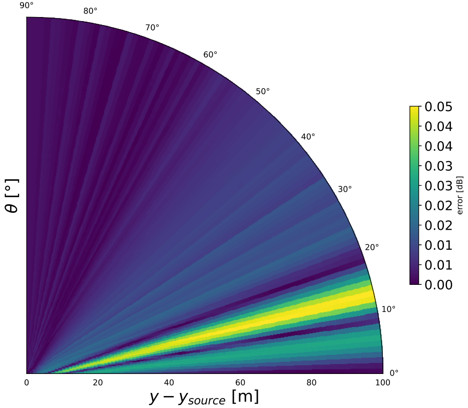
Free Field - pulse



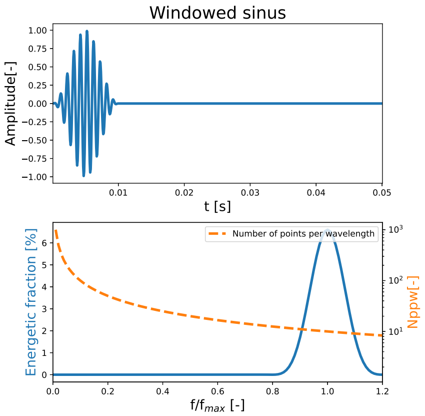
Error on group speed, $\Delta\ell = \lambda/10$, $f_{\max} = 2400$ Hz.

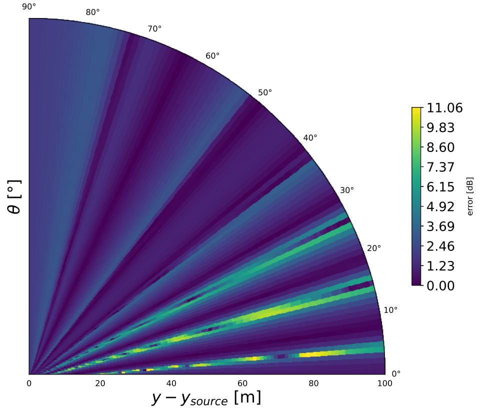
Ground reflection - pulse



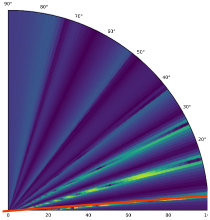


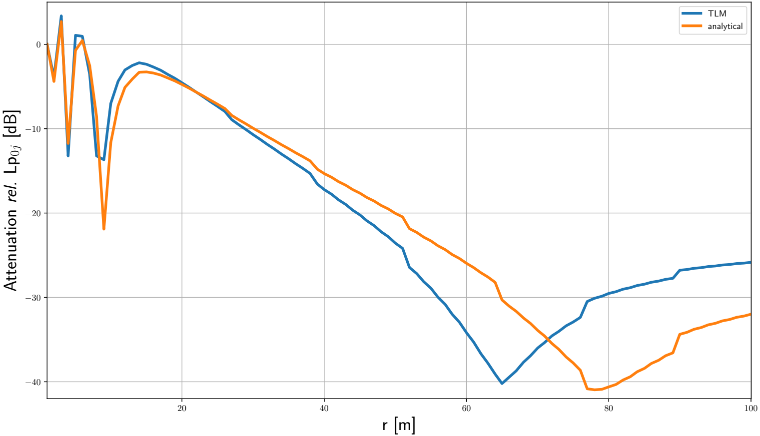
Ground reflection - windowed sinus



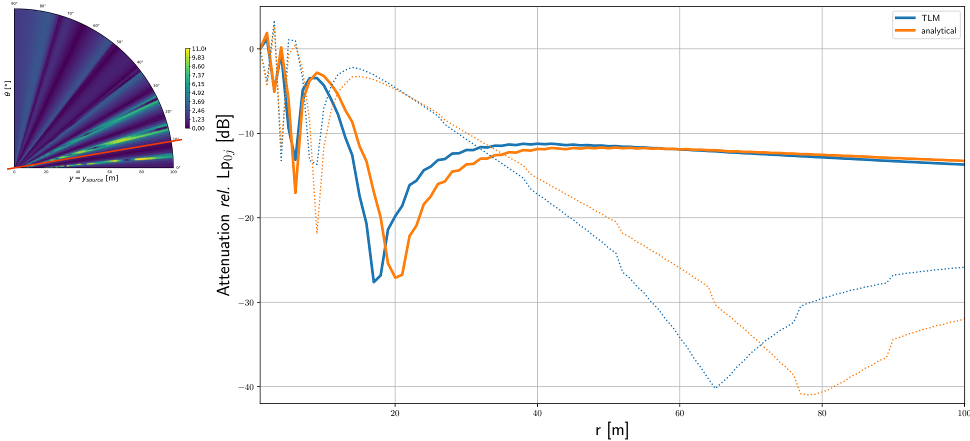


Ground reflection - windowed sinus, $\theta = 4^\circ$



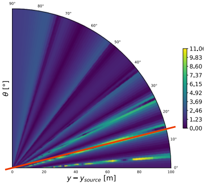


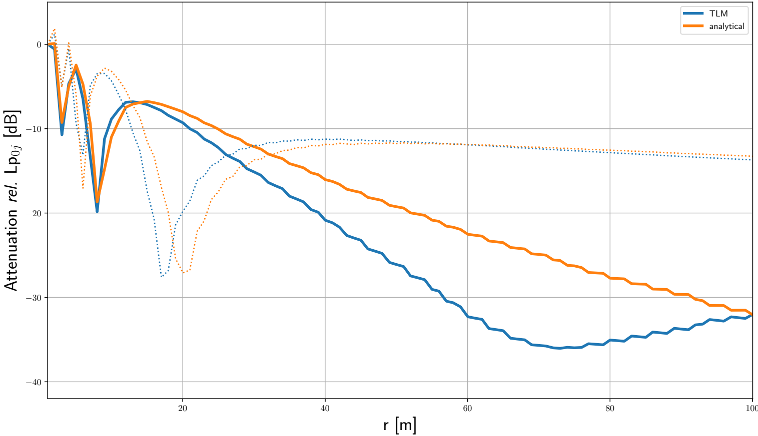
Ground reflection - windowed sinus, $\theta = 10^\circ$



$\theta = 4^\circ$

Ground reflection - windowed sinus, $\theta = 15^\circ$

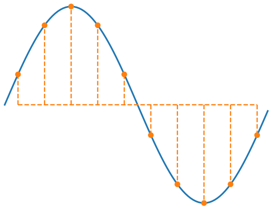




$\theta = 10^\circ$

Conclusion & perspectives

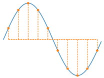
◦ 10 points per wavelength criterion not sufficient

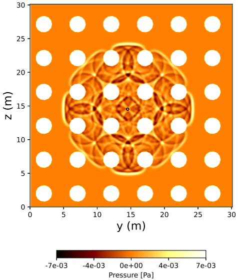


Conclusion & perspectives

- 10 points per wavelength criterion not sufficient

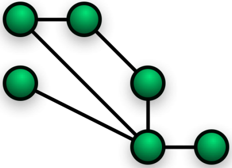
- Compare complex modelling with measurements

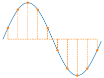


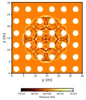


- 10 points per wavelength criterion not sufficient

- Compare complex modelling with measurements
- Increase the scheme order or change?







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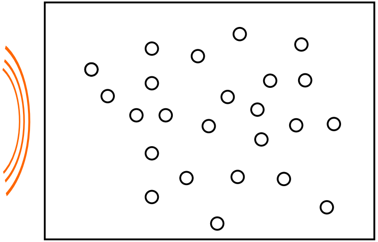
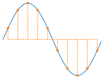
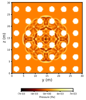
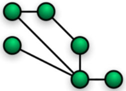
- 10 points per wavelength criterion not sufficient

- Compare complex modelling with measurements
- Increase the scheme order or change?
- Waves in complex media theory









Conclusion & perspectives

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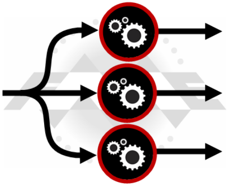
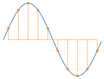
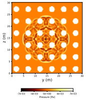
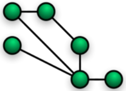
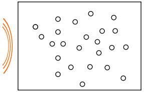
- Increase the scheme order or change?
- Waves in complex media theory
- MPI parallelization











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Thank you for listening!

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- links :
  - `http://www.umrae.fr/`

SCAN ME

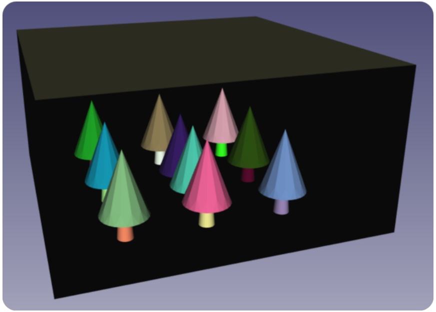
Unité Mixte de Recherche en Acoustique Environnementale (UMRAE) is a joint research unit in environmental acoustics between *Gustave Eiffel University & Cerema*.

Appendix

Problem & method used

Meshing & implementation

- FreeCAD API automatic scene generation
  - (Supervision of an IT student's internship)



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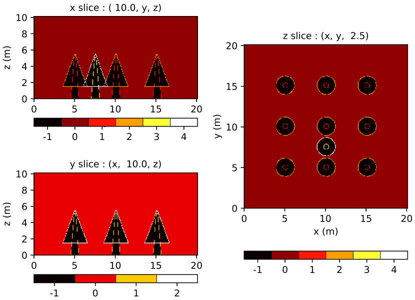
  `https://github.com/nicolas-f/FastVoxel`



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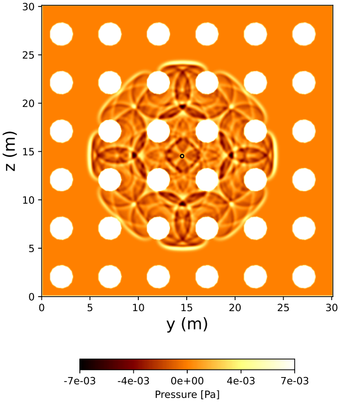
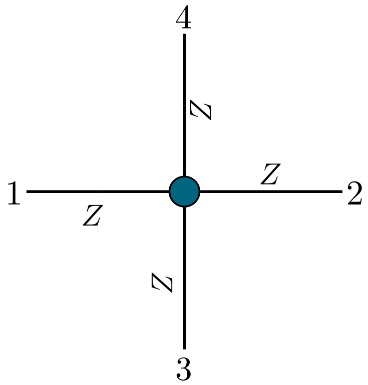
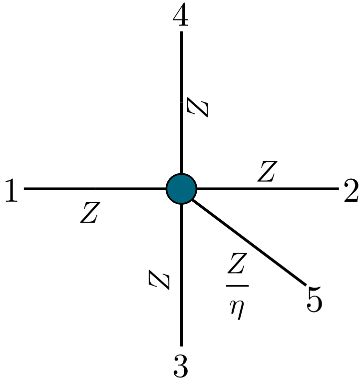
- TLM solver



Inhomogeneous TLM - principle



2D stencil around a node



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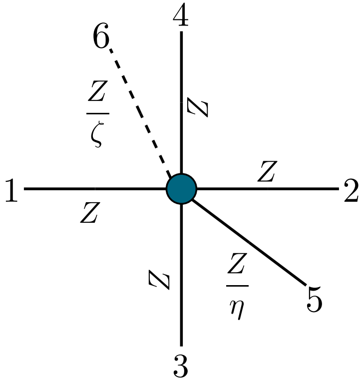

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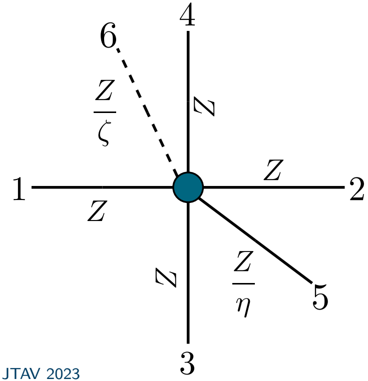


Link to wave equation

$$\frac{1}{c_{\text{TLM}}^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2 p + \frac{\zeta_{(j,l)}}{2\Delta\ell^2}\left(2\Delta t\frac{\partial p}{\partial t}\right) = \mathcal{O}\left(\frac{\Delta t^4}{\Delta\ell^2}\right) + \mathcal{O}\left(\frac{\Delta t^3}{\Delta\ell^2}\right) + \mathcal{O}\left(\Delta\ell^2\right) \quad (4)$$

Inhomogeneous TLM - analysis

Stability

$$c_{\text{TLM}}\left(\mathbf{x_r}\right) = \sqrt{\frac{2}{\eta_{\mathbf{r}} + 2d}\frac{\Delta\ell}{\Delta t}} \tag{5}$$

$$\eta \geq 0$$

Dispersion

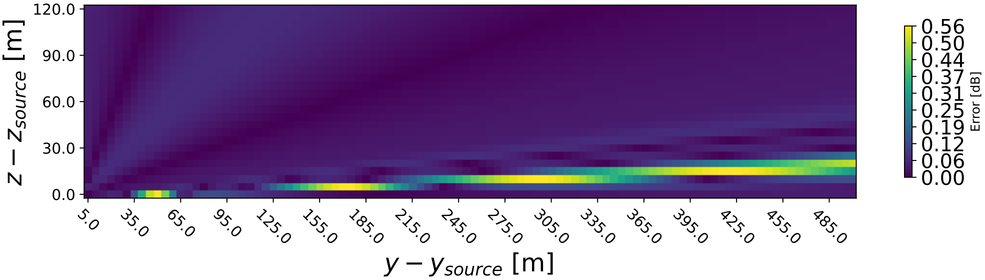
$$\|\underline{k}\| \approx k_{0\text{TLM}} + i\alpha_{\text{TLM}} + \mathcal{O}(\zeta^2) \tag{6}$$

$$k_{0\text{TLM}} = \frac{\omega}{c_{\text{TLM}}} \text{ and } \qquad \alpha_{\text{TLM}} = \frac{\zeta}{\sqrt{2(\eta + 2d)}\Delta\ell}$$

Current results

Ground reflection : Richer Wavelet

Error map: $f_{\max} = 2000$ Hz, $r_{\max} = 500$ m et $h_{\text{src}} = 2$ m



Ground reflection : windowed sinus

Error map: $f_{max} = 2000$ Hz, $r_{max} = 500$ m et $h_{src} = 2$ m