Modélisation numérique de la propagation des ondes élastiques guidées dans les câbles hélicoïdaux précontraints

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1 Introduction
   • Context
   • Structural complexity
   • Experimental results

2 Helical geometry

3 Interwire coupling

4 Prestress

5 Excitation

6 Conclusion
Application of guided waves to cables

Application of guided waves to cables

NDT and SHM of bridges: damage detection, tension estimation,…

Potentiality:
- propagation over long distances
- sensitivity to small damages

Figure: **Left:** anchorage zone (single access ⇒ reflectometry), **right:** 6+1 strand.
Context

Application of guided waves to cables

NDT and SHM of bridges: damage detection, tension estimation,...

Potentiality:
- propagation over long distances
- sensitivity to small damages

Drawbacks:
- dispersive
- multimodal

Figure: Left: anchorage zone (single access ⇒ reflectometry), right: 6+1 strand.

Complex physics

Modeling is essential to further optimize inspection techniques
Figure: Complicating effects: geometry curvature
Figure: Complicating effects: geometry curvature, interwire coupling, prestress, embedment
Some experimental results at IFSTTAR (Laguerre et al., BLPC, 2002)

**Figure:** Longitudinally-polarized magnetostrictive device

**Figure:** Experimental spectrogram (time-frequency) for a $6+1$ strand under tensile load: 2%, 10% and 60% of UTS

Missing frequency $\sim 67$kHz ('notch', Kwun et al., 1998)
Some experimental results at IFSTTAR (Laguerre et al., BLPC, 2002)

Figure: Longitudinally-polarized magnetostrictive device

Increase of notch frequency from 67kHz to 88kHz

Missing frequency $\sim 67$kHz ('notch', Kwun et al., 1998)

Figure: Experimental spectrogram (time-frequency) for a 6+1 strand under tensile load: 2%, 10% and 60% of UTS
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Do guided waves exist in curved structures? (Treyssède, JASA, 2011)

Basic assumption for guided waves: \( \propto e^{iks} \times e^{-i\omega t} \)

- \( k \): axial wavenumber, \( \omega \): angular frequency

Where does this exponential dependence come from?

Separation of variables: coefficients of differential equations must not depend on \( s \)!

Translational invariance for curved waveguides

- cross-section ✓
- physical properties ✓

\(^a\) For elastodynamics, the strain operator: \( \frac{1}{2}(\nabla(\cdot) + \nabla^T(\cdot)) \)
Do guided waves exist in curved structures? (Treyssède, JASA, 2011)

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Translational invariance for curved waveguides

- cross-section ✓
- physical properties ✓
- coeff. of basic operators$^a$ must not depend on $s$ $\rightarrow$ ?

$^a$for elastodynamics, the strain operator: $\frac{1}{2} \left( \nabla(\cdot) + \nabla^T(\cdot) \right)$

Answer to point 3: calculate the METRIC TENSOR of the curvilinear coordinate system and show it is not dependent on $s$

$\rightarrow$ differential geometry and tensor analysis required...
The helical coordinate system \((x, y, s)\):

- \(R(s)\) is a helix curve (constant curvature \(\kappa\) and torsion \(\tau\))
- \(X(x, y, s) = R(s) + xN(s) + yB(s)\)

**Figure:** One helix step, 
\((N, B, T)\): Serret-Fr. basis
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**Definitions:**

- covariant basis: \((g_1, g_2, g_3) = (dX/dx, dX/dy, dX/ds)\)
- metric tensor: \((g)_{ij} = g_i \cdot g_j\)

**Main results**

- proof of translational invariance (metric tensor):
  \[
  g = \begin{bmatrix}
  1 & 0 & -\tau y \\
  0 & 1 & \tau x \\
  -\tau y & \tau x & \tau^2(x^2 + y^2) + (1 + \kappa x)^2
  \end{bmatrix}
  \]

- Strain operator \(\epsilon = (L_{xy} + L_s \partial/\partial s)u:\)

\[
L_{xy} = \begin{bmatrix}
(1 + \kappa x)\partial/\partial x & 0 \\
0 & (1 + \kappa x)\partial/\partial y \\
\kappa & 0 \\
\tau y \partial/\partial x - \tau x \partial/\partial y & 0 \\
\tau y \partial/\partial x - \tau x \partial/\partial y & (1 + \kappa x)\partial/\partial x \\
\tau x \partial/\partial y & (1 + \kappa x)\partial/\partial y
\end{bmatrix}
\]

\[
L_s = \frac{1}{1 + \kappa x} \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0 \\
1 & 0 & 0 \\
0 & 1 & 0
\end{bmatrix}
\]

**Figure:** One helix step, \((N, B, T)\): Serret-Fr. basis
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   - Numerical method
   - Results for typical 6+1 strands

4 Prestress

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**Simplifying assumption:** stick contact conditions *(no slip, no separation, no friction)*

**What is the appropriate coordinate system?**

- **central wire:** fixed cartesian coordinate system \((\kappa, \tau) = (0, 0)\)
- **peripheral wires:** helical coord. systems with same \((\kappa, \tau)\) BUT based on different helix centrelines
  → problem: coordinate systems are all different...
- **6+1 strand:** ?
**On the translational invariance of 6+1 strands** (Treyssède et al., JSV, 2010)

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- **6+1 strand**: **UNIFYING SYSTEM = TWISTING**

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<th>Cartesian system</th>
<th>Straight wire</th>
<th>Helical wire</th>
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<td>((\kappa = \tau = 0))</td>
<td>3 conditions satisfied</td>
<td>condition 1 not satisfied</td>
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| Helical system \((\kappa \neq 0, \tau \neq 0)\) | condition 1:? |

| Twisting system \((\kappa = 0, \tau = 2\pi / L)\) | 3 conditions satisfied | 3 conditions satisfied |

**Conditions for invariance:**
- cross-section →?
- physical properties ✓
- coefficients not fct of \(s\) ✓
Variational formulation for 3D elastodynamics:

\[ \int_{\Omega} \delta \epsilon^T C \epsilon dV + \int_{\Omega} \rho \delta u^T \ddot{u} dV = 0, \quad \text{where} \; \epsilon = (L_{xy} + L_s \partial / \partial s)u \]

Assume:

\[ u(x, y, s, t) = u(x, y)e^{i(ks - \omega t)} \]

Quadratic eigenvalue problem (after FE discretisation)

\[ \{K_1 - \omega^2 M + ik(K_2 - K_2^T) + k^2 K_3\} U = 0 \]

- problem reduced on the cross-section only
- solved for each frequency \( \omega \)
- for a given \( \omega \), solution = guided modes \( (k_n^{\pm}, U_n^{\pm}) \)
Numerical results for typical 6+1 strands

**Numerical method:** Semi-Analytical Finite Element method

Cross-section FE mesh for the seven-wire strand ($\phi = 7.9^\circ$):
- peripheral wires do not touch each other
- cross-sections are not circular in the twisted system
- mesh refinement at contact points (6975 dofs, here)

**Figure:** Dimensionless energy velocity vs. frequency ([0;2]~[0;400]kHz). **Right:** seven-wire strand.
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**Figure:** Dimensionless energy velocity vs. frequency ([0;2]≈[0;400]kHz). **Right:** seven-wire strand. **Left:** single wires (black: central, gray: peripheral).
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Equilibrium equations = elastodynamics updated on the deformed geometry with an additional term (geometric stiffness):

\[
\int_{\Omega_0} \delta \epsilon^T C \epsilon \, dV + \int_{\Omega_0} \text{tr}(\nabla \delta u \cdot \sigma_0 \cdot \nabla u^T) \, dV + \int_{\Omega_0} \rho \delta u^T \ddot{u} \, dV = 0
\]

Prestress state needed: deformed section \( S_0 \) and Cauchy stress \( \sigma_0 \) (transl. inv.!) → development of homogenization method for the statics of cables: \( K_1 U_m = F_\epsilon \)
Including prestressing effects (Collaboration with GeM, Frikha, PhD thesis, 2010)

Equilibrium equations = elastodynamics updated on the deformed geometry with an additional term (geometric stiffness):

$$\int_{\Omega_0} \delta \epsilon^T C \epsilon dV + \int_{\Omega_0} \text{tr}(\nabla \delta \mathbf{u} \cdot \sigma_0 \cdot \nabla \mathbf{u}^T) dV + \int_{\Omega_0} \rho \delta \mathbf{u}^T \ddot{\mathbf{u}} dV = 0$$

Prestress state needed: deformed section $S_0$ and Cauchy stress $\sigma_0$ (transl. inv.) → development of homogenization method for the statics of cables: $K_1 \mathbf{U}_m = F_\epsilon$

**Figure:** Numerical results. Left: without axial load, right: with axial load ($\epsilon = 0.6\%$)
And now let’s account for contact...

- Contact procedure: iterative, node-to-node
- Displacement continuity by direct elimination method

![Numerical results](image)

**Figure:** Numerical results. Left: without axial load, right: with axial load ($\epsilon = 0.6\%$)

- The notch frequency rises from 65 kHz to 86 kHz, ~as in the experiments
- Origin of phenomenon: increase of contact area as the tensile load increases
- With the prediction model, significant effects are observed for other modes
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SAFE under excitation: \( \{K_1 - \omega^2 M + ik(K_2 - K_2^T) + k^2 K_3\} U(k; \omega) = F(k; \omega) \)

Modal expansion in the Fourier domain: \( U(k; \omega) = \sum_{m=1}^{M} \beta_m(k; \omega) U_m(\omega) \)

SAFE biorthogonality + residue calculus:

Response under excitation

\[
U(z; \omega) = \sum_{m=1}^{M} \alpha_m(\omega) \frac{U_m(\omega)}{\sqrt{P_m(\omega)}} e^{ik_m(\omega)z}
\]

with:

\[
\alpha_m(\omega) = \frac{i\omega}{4\sqrt{P_m(\omega)}} U^*_m(\omega) F(k_m; \omega)
\]

\( P_m(\omega) \): power flow of the \( m \)th mode

Symplifying assumption: propagating modes only \((k \in \mathbb{R}, \text{no viscoelasticity})\)
SAFE biorthogonality \( \equiv \) discretized version of Auld’s complex orthogonality (Treyssède et al. JASA 2013)
Energy transfer from central wire

Excitation at $z = 0$, distributed over the central wire, oriented along the axis → compressional modes, mainly

Power flow ratio

$$\eta = 1 - \frac{P_{\text{central}}}{P_{\text{total}}}$$

$P$: power flow of the excited field (multimodal)

warning: $P$ of a subsystem oscillates along $z$ due to multimodal propagation (so does $\eta$)
Energy transfer from central wire

Excitation at $z = 0$, distributed over the central wire, oriented along the axis $\rightarrow$ compressional modes, mainly

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Figure: $\eta$ as a function of frequency for a strand, unloaded (left) and under 0.6% tensile strain (right). $\eta$ is $z$-averaged for each frequency.
What happens?

Modal coefficients (left) and energy velocity (right) under 0.6% tensile strain

$L'(0, 1)$ mode shape at $\omega/c_s = 1.6$
What happens?

Modal coefficients (left) and energy velocity (right) under 0.6% tensile strain

\( L'(0, 1) \) mode shape at \( \omega / c_s = 1.6 \)

Unloaded case
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Embedment modeling: **SAFE-PML** formulation, collaboration with ENSTA (K. L. Nguyen, PhD student, 2011-2014) - not shown here for conciseness

Still some works!...

- experimental validation (only done for \( L(0, 1) \))
- numerical improvements (high frequency FE limitation, ...)
- ...

Development of a generic in-house FE code dedicated to waveguides:

- guided modes in complex waveguides (2D, 3D, anisotropic, prestressed, curved, ...)
- response under excitation (**bi-orthogonality**)
- scattering by inhomogeneities (**hybrid FE-SAFE method**)

Other applications: acoustic emission, composites, geosciences...
Merci de votre attention

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Validation tests

- **Analytical: helical Timoshenko beam model** (Wittrick, 1966)
  - six degrees of freedom (3 displacements + 3 rotations)
  - limitations: only valid for $(\kappa a)^2 \ll 1$ and low frequencies $(\omega a/c_s \leq 2)$.

- **Numerical: periodic FE element, based on Floquet boundary conditions**
  - reference model: one complete helix step (Treysse, JASA, 2007)
  - limitations: no, except the size of FE matrices...

**Figure:** Mesh of a complete step for a helical waveguide $\phi = 45^\circ$ (inset: one layer mesh). Periodic Floquet boundary conditions are applied at the inlet and outlet of waveguide.
Figure: FE mesh for a helical waveguide $R = 2a$, $\phi = 45^\circ$. **Left:** analysis with $(\kappa a, \tau a) = (1/4, 1/4)$, **right:** analysis with the twisting coordinate system $(\kappa a, \tau a) = (0, 1/2)$. 
A validation test for twist

Figure: FE mesh for a helical waveguide $R = 2a$, $\phi = 45^\circ$. **Left:** analysis with $(\kappa a, \tau a) = (1/4, 1/4)$, **right:** analysis with the twisting coordinate system $(\kappa a, \tau a) = (0, 1/2)$.

Figure: $\omega a/c_s$ vs. $ka$ for $R = 2a$, $\phi = 45^\circ$. **Black:** helical system (reference), **gray:** twisting.

$k_t = k_h \frac{1}{L}$
A validation test for twist

Figure: FE mesh for a helical waveguide $R = 2a$, $\phi = 45^\circ$. **Left:** analysis with $(\kappa a, \tau a) = (1/4, 1/4)$, **right:** analysis with the twisting coordinate system $(\kappa a, \tau a) = (0, 1/2)$.

Figure: $\omega a/c_s$ vs. $ka$ for $R = 2a$, $\phi = 45^\circ$. **Black:** helical system (reference), **gray:** twisting.

Figure: The cylinder case. **Black:** cartesian system (reference), **gray:** twisting.
L(0,1) compressional-like mode

Figure: **Single wires. Up:** central, **down:** peripheral. x-marks: experimental group velocities, solid lines: SAFE energy velocities.
Comparison with experiments

**L(0,1)** compressional-like mode

![Graph showing comparison between experimental and SAFE energy velocities.](image)

**Figure:** Single wires. **Up:** central, **down:** peripheral. x-marks: experimental group velocities, solid lines: SAFE energy velocities.

**Figure:** 6+1 strand: experimental spectrogram (time-frequency). Gray solid lines: SAFE predictions of group-time delays.

'Notch' frequency at 65kHz = curve veering between two compressional modes.
Notch frequency: a curve veering phenomenon

'Curve veering': encountered in eigenvalue problems of weakly coupled systems (Perkins et al. 86, Pierre 88, JSV)

Figure: Repulsion of two distinct branches, veering away from each other at 0.35

- Branches 1 and 2 have interchanged their properties (wavenumber, modeshares)
- Both have a global motion of compressional type

Figure: From left to right: real part (top) and imaginary part (bottom) of the displacement modeshape of branch 1 computed at points 1a, 1b and 1c.

Figure: Modeshape of branch 2 computed at points 2a, 2b and 2c.
Let’s change the excitation...

**Excitation along** $z$ **of a peripheral** wire

**Flexural** excitation of central wire ($x$ direction)

**Torsional** excitation of central wire → existence of a $T'(0, 1)$ mode