JOURNEES TECHNIQUES ACOUSTIQUE ET VIBRATIONS

Modélisation numérique de la propagation des ondes élastiques guidées dans les câbles hélicoïdaux précontraints

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Contents

Context

Application of guided waves to cables

NDT and SHM of bridges: damage detection, tension estimation,...

Potentiality:

- propagation over long distances
- sensitivity to small damages

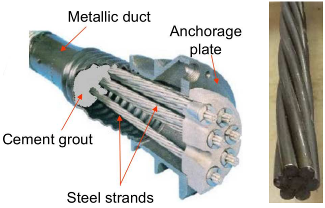

Figure: **Left:** anchorage zone (single access⇒reflectometry), **right:** 6+1 strand.

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Application of guided waves to cables

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Drawbacks:

- dispersive
- multimodal

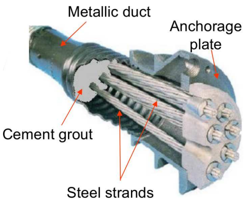

Figure: **Left:** anchorage zone (single access⇒reflectometry), **right:** 6+1 strand.

Complex physics

Modeling is essential to further optimize inspection techniques

Structural Complexity

Figure: Complicating effects: geometry curvature

Structural Complexity

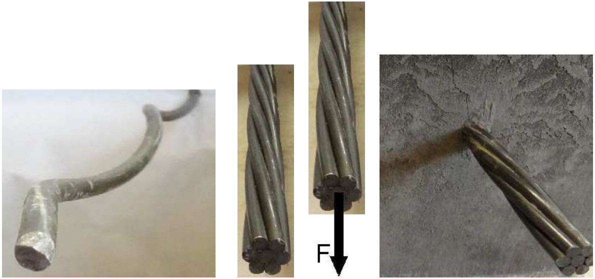

Figure: Complicating effects: geometry curvature, interwire coupling, prestress, embedment

Some experimental results at IFSTTAR (Laguerre et al., BLPC, 2002)

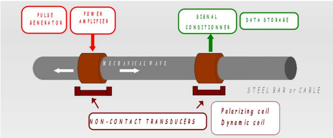

Figure: Longitudinally-polarized magnetostrictive device

Missing frequency ∼67kHz ('notch', Kwun et al., 1998)

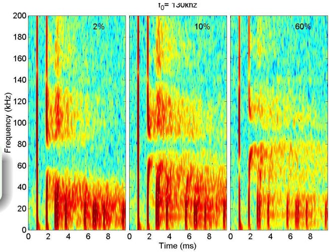

Figure: Experimental spectrogram (time-frequency) for a **6+1 strand** under tensile load: 2%, 10% and 60% of UTS

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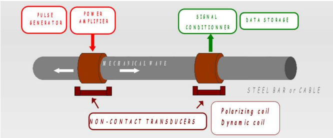

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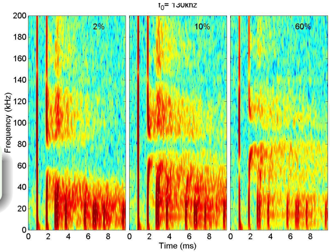

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Increase of notch frequency from 67kHz to 88kHz

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Do guided waves exist in curved structures? (Treyssède,JASA,2011)

Basic assumption for guided waves: $\propto e^{iks} \quad (\times e^{-i\omega t})$

k: axial wavenumber, ω: angular frequency

Where does this exponential dependence come from?

Separation of variables: **coefficients of differential equations must not depend on s!**

Translational invariance for curved waveguides

1. cross-section ✓
2. physical properties ✓

[a] for elastodynamics, the strain operator: $\frac{1}{2}(\nabla(\cdot) + \nabla^T(\cdot))$

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Translational invariance for curved waveguides

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2. physical properties ✓
3. **coeff. of basic operators[a] must not depend on s** $\rightarrow$?

[a] for elastodynamics, the strain operator: $\frac{1}{2}(\nabla(\cdot) + \nabla^T(\cdot))$

Answer to point 3: calculate the **METRIC TENSOR** of the curvilinear coordinate system and show it is not dependent on s

$\rightarrow$ **differential geometry and tensor analysis** required...

Existence of guided modes in helical waveguides (Treyssède, Wave Motion, 2008)

The helical coordinate system (x, y, s):

- $\mathbf{R}(s)$ is a helix curve (constant curvature κ and torsion τ)
- $\mathbf{X}(x, y, s) = \mathbf{R}(s) + x\mathbf{N}(s) + y\mathbf{B}(s)$

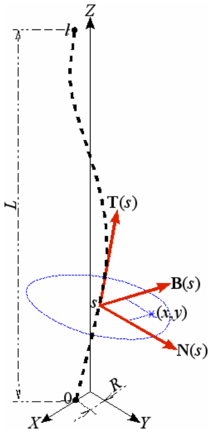

Figure: One helix step, $(\mathbf{N}, \mathbf{B}, \mathbf{T})$: Serret-Fr. basis

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Definitions:

- covariant basis: $(\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3) = (d\mathbf{X}/dx, d\mathbf{X}/dy, d\mathbf{X}/ds)$
- metric tensor: $(\mathbf{g})_{ij} = \mathbf{g}_i \cdot \mathbf{g}_j$

Main results

- proof of translational invariance (metric tensor):

$$\mathbf{g} = \begin{bmatrix} 1 & 0 & -\tau y \\ 0 & 1 & \tau x \\ -\tau y & \tau x & \tau^2(x^2+y^2) + (1+\kappa x)^2 \end{bmatrix}$$

- Strain operator $\boldsymbol{\epsilon} = (\mathbf{L}_{xy} + \mathbf{L}_s \partial/\partial s)\mathbf{u}$:

$$\mathbf{L}_{xy} = \frac{1}{1+\kappa x}\begin{bmatrix} (1+\kappa x)\partial/\partial x & 0 & 0 \\ 0 & (1+\kappa x)\partial/\partial y & 0 \\ \kappa & 0 & \tau y\partial/\partial x - \tau x\partial/\partial y \\ (1+\kappa x)\partial/\partial y & (1+\kappa x)\partial/\partial x & 0 \\ \tau y\partial/\partial x - \tau x\partial/\partial y & -\tau & -\kappa + (1+\kappa x)\partial/\partial x \\ \tau & \tau y\partial/\partial x - \tau x\partial/\partial y & (1+\kappa x)\partial/\partial y \end{bmatrix}$$

$$\mathbf{L}_s = \frac{1}{1+\kappa x}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

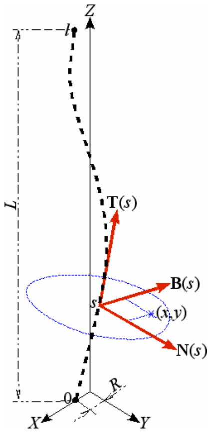

Figure: One helix step, $(\mathbf{N}, \mathbf{B}, \mathbf{T})$: Serret-Fr. basis

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On the translational invariance of 6+1 strands (Treyssède et al., JSV, 2010)

Simplifying assumption: stick contact conditions (**no slip, no separation, no friction**)

What is the appropriate coordinate system?

- **central wire:** fixed cartesian coordinate system $(\kappa, \tau) = (0, 0)$
- **peripheral wires:** helical coord. systems with same (κ, τ) BUT based on different helix centrelines
  $\longrightarrow$problem: coordinate systems are all different...
- **6+1 strand:** ?

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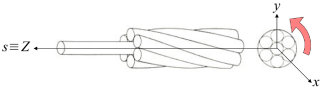
- **6+1 strand: UNIFYING SYSTEM = TWISTING**

	Straight wire	**Helical wire**
Cartesian system $(\kappa=\tau=0)$	3 conditions satisfied	condition 1 not satisfied
Helical system $(\kappa\neq0, \tau\neq0)$	condition 1 :?	
Twisting system $(\kappa=0, \tau=2\pi/L)$	3 conditions satisfied	3 conditions satisfied



Conditions for invariance:

1. cross-section $\longrightarrow$?
2. physical properties ✓
3. coefficients not fct of s ✓

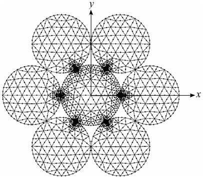
Semi-Analytical Finite Element method

Variational formulation for 3D elastodynamics:

$$\int_\Omega \delta\epsilon^T \mathbf{C}\epsilon dV + \int_\Omega \rho\delta\mathbf{u}^T \ddot{\mathbf{u}} dV = 0, \quad \text{where } \boldsymbol{\epsilon} = (\mathbf{L}_{xy} + \mathbf{L}_s\partial/\partial s)\mathbf{u}$$

Assume:

$$\mathbf{u}(x,y,s,t) = \mathbf{u}(x,y)e^{i(ks-\omega t)}$$



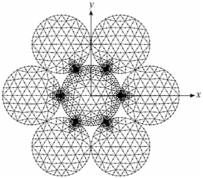
Quadratic eigenvalue problem (after FE discretisation)

$$\{\mathbf{K}_1 - \omega^2\mathbf{M} + ik(\mathbf{K}_2 - \mathbf{K}_2^T) + k^2\mathbf{K}_3\}\mathbf{U} = \mathbf{0}$$

- problem reduced on the cross-section only
- solved for each frequency ω
- for a given ω, solution = guided modes $(k_n^\pm, \mathbf{U}_n^\pm)$

Numerical results for typical 6+1 strands

Numerical method: **S**emi-**A**nalytical **F**inite **E**lement method



Cross-section FE mesh for the seven-wire strand ($\phi = 7.9°$):
- peripheral wires do not touch eachother
- cross-sections are not circular in the twisted system
- mesh refinement at contact points (6975 dofs, here)

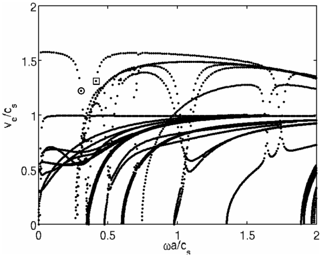


Figure: Dimensionless energy velocity vs. frequency ([0;2]≈[0;400]kHz). **Right:** seven-wire strand.

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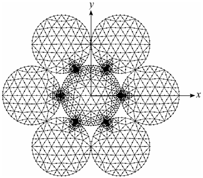

velocity drop of 'L(0,1)' at 0.33=65kHz

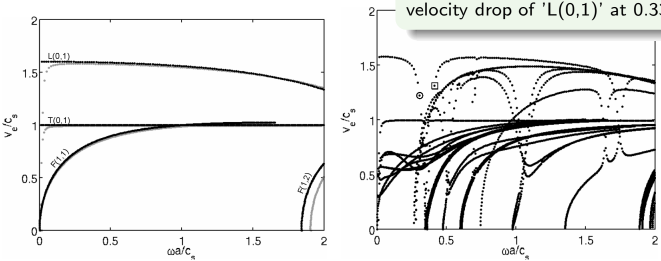

Figure: Dimensionless energy velocity vs. frequency ([0;2]≈[0;400]kHz). **Right:** seven-wire strand. **Left:** single wires (black: central, gray: peripheral).

Contents

Including prestressing effects (Collaboration with GeM, Frikha, PhD thesis, 2010)

- Equilibrium equations = elastodynamics updated on the deformed geometry with an additional term (geometric stiffness):

$$\int_{\Omega_0} \delta\boldsymbol{\epsilon}^T \mathbf{C}\boldsymbol{\epsilon} dV + \int_{\Omega_0} \mathrm{tr}(\nabla\delta\mathbf{u}\cdot\boldsymbol{\sigma}_0\cdot\nabla\mathbf{u}^T)dV + \int_{\Omega_0} \rho\delta\mathbf{u}^T\ddot{\mathbf{u}}dV = 0$$

- Prestress state needed: deformed section S_0 and Cauchy stress σ_0 (transl. inv.!) $\rightarrow$ development of homogenization method for the statics of cables: $\mathbf{K}_1\mathbf{U}_m = \mathbf{F}_\epsilon$

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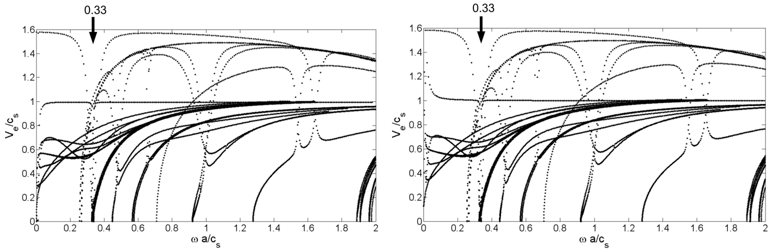
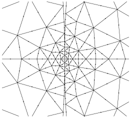
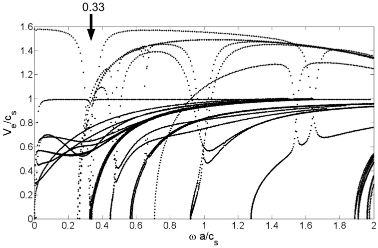


Figure: Numerical results. Left: without axial load, right: with axial load ($\epsilon = 0.6\%$)

And now let's account for contact...

- Contact procedure: iterative, node-to-node
- Displacement continuity by direct elimination method





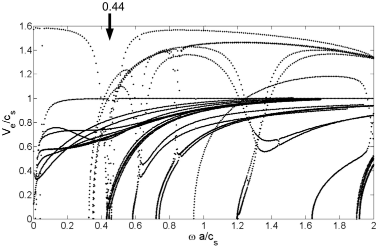


Figure: Numerical results. Left: without axial load, right: with axial load ($\epsilon = 0.6\%$)

- The notch frequency rises from 65 kHz to 86 kHz, ∼as in the experiments
- Origin of phenomenon: increase of contact area as the tensile load increases
- With the prediction model, significant effects are observed for other modes

Contents

Modal expansion

- SAFE under excitation: $\{\mathbf{K}_1 - \omega^2\mathbf{M} + ik(\mathbf{K}_2 - \mathbf{K}_2^T) + k^2\mathbf{K}_3\}\mathbf{U}(k;\omega) = \mathbf{F}(k;\omega)$
- Modal expansion in the Fourier domain: $\mathbf{U}(k;\omega) = \sum_{m=1}^{M} \beta_m(k;\omega)\mathbf{U}_m(\omega)$
- SAFE biorthogonality + residue calculus:

Response under excitation

$$\mathbf{U}(z;\omega) = \sum_{m=1}^{M} \alpha_m(\omega)\frac{\mathbf{U}_m(\omega)}{\sqrt{P_m(\omega)}}e^{ik_m(\omega)z}$$

with:

$$\alpha_m(\omega) = \frac{i\omega}{4\sqrt{P_m(\omega)}}\mathbf{U}_m^*(\omega)\mathbf{F}(k_m;\omega)$$

$P_m(\omega)$: power flow of the mth mode

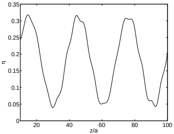
Symplifying assumption: propagating modes only ($k \in \mathbb{R}$, no viscoelasticity)
SAFE biorthogonality $\equiv$ discretized version of Auld's complex orthogonality (Treyssède et al. JASA 2013)

Energy transfer from central wire

Excitation at $z = 0$, distributed over the central wire, oriented along the axis
$\rightarrow$ compressional modes, mainly

Power flow ratio

$\eta = 1 - \frac{P_{\text{central}}}{P_{\text{total}}}$
P: power flow of the excited field (multimodal)



warning: P of a subsystem oscillates along z due to multimodal propagation (so does η)

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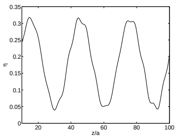

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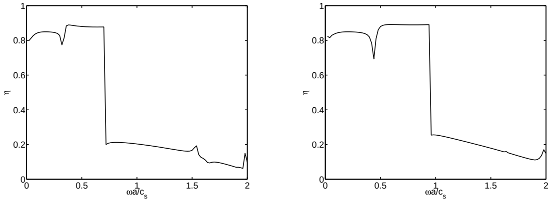
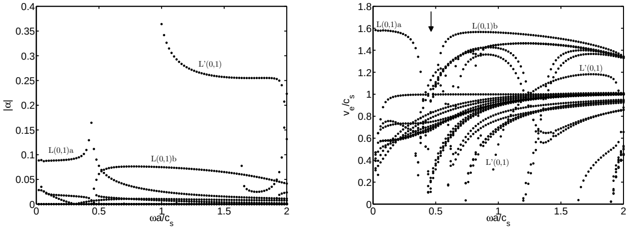
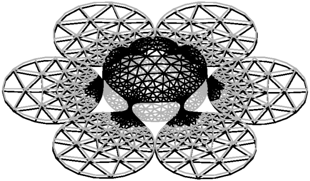


Figure: η as a function of frequency for a strand, unloaded (left) and under 0.6% tensile strain (right). η is z-averaged for each frequency.

What happens?



Modal coefficients (left) and energy velocity (right) under 0.6% tensile strain



$L'(0,1)$ mode shape at $\omega/c_s = 1.6$

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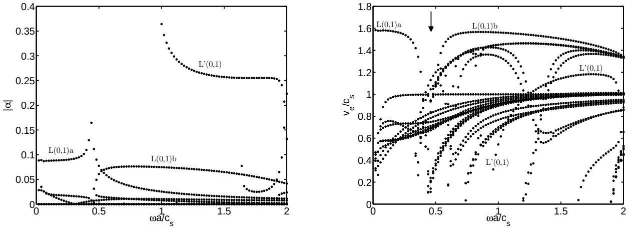

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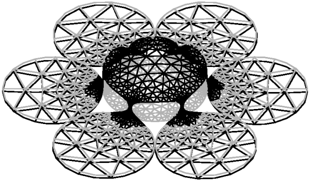

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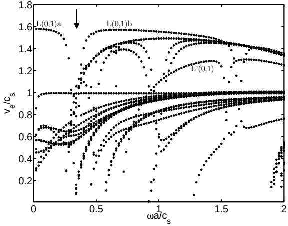

Unloaded case

Contents

Conclusion

Embedment modeling: **SAFE-PML** formulation, collaboration with ENSTA (K. L. Nguyen, PhD student, 2011-2014) - not shown here for conciseness

Still some works!...

- experimental validation (only done for $L(0, 1)$)
- numerical improvements (high frequency FE limitation,...)
- ...

Development of a generic in-house FE code dedicated to waveguides:

- guided modes in complex waveguides (2D, 3D, anisotropic, prestressed, curved,...)
- response under excitation (**bi-orthogonality**)
- scattering by inhomogeneities (**hybrid FE-SAFE method**)

Other applications: acoustic emission, composites, geosciences...

Merci de votre attention

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Validation tests

- **Analytical: helical Timoshenko beam model** (Wittrick, 1966)
  - six degrees of freedom (3 displacements + 3 rotations)
  - limitations: only valid for $(\kappa a)^2 \ll 1$ and low frequencies ($\omega a/c_s \leq 2$).
- **Numerical: periodic FE element, based on Floquet boundary conditions**
  - reference model: one complete helix step (Treyssede, JASA, 2007)
  - limitations: no, except the size of FE matrices...

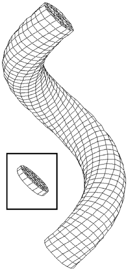

Figure: Mesh of a complete step for a helical waveguide $\phi = 45^\circ$ (inset: one layer mesh). Periodic Floquet boundary conditions are applied at the inlet and outlet of waveguide.

A validation test for twist

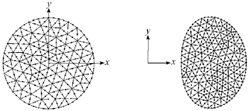

Figure: FE mesh for a helical waveguide $R = 2a$, $\phi = 45^\circ$. **Left:** analysis with $(\kappa a, \tau a) = (1/4, 1/4)$, **right:** analysis with the twisting coordinate system $(\kappa a, \tau a) = (0, 1/2)$.

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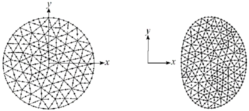

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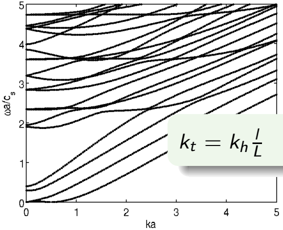

$$k_t = k_h \frac{l}{L}$$

Figure: $\omega a / c_s$ vs. ka for $R = 2a$, $\phi = 45^\circ$.
Black: helical system (reference), **gray:** twisting.

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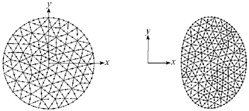

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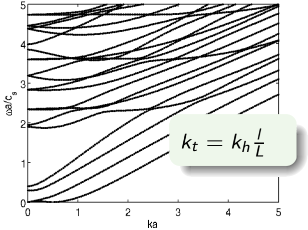

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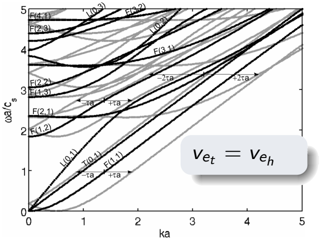


Figure: **The cylinder case**. **Black:** cartesian system (reference), **gray:** twisting.

Comparison with experiments

L(0,1) compressional-like mode

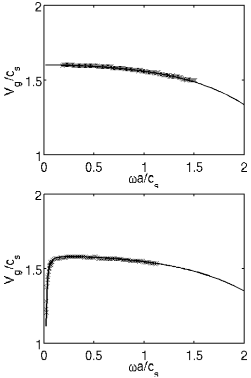


Figure: **Single wires**. **Up:** central, **down:** peripheral. x-marks: experimental group velocities, solid lines: SAFE energy velocities.

Comparison with experiments

L(0,1) compressional-like mode

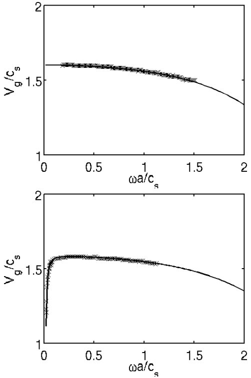


Figure: **Single wires**. **Up:** central, **down:** peripheral. x-marks: experimental group velocities, solid lines: SAFE energy velocities.

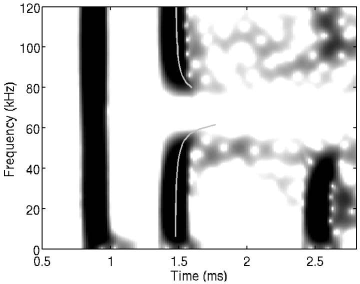


Figure: **6+1 strand:** experimental spectrogram (time-frequency). Gray solid lines: SAFE predictions of group-time delays.

'Notch' frequency at 65kHz = curve veering between two compressional modes

Notch frequency: a curve veering phenomenon

'Curve veering': encountered in eigenvalue problems of weakly coupled systems (Perkins et al. 86, Pierre 88, JSV)

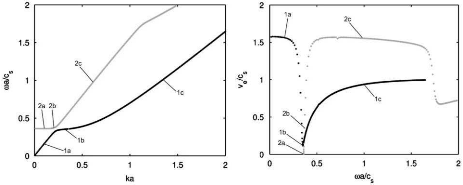


Figure: Repulsion of two distinct branches, veering away from each other at 0.35

- Branches 1 and 2 have interchanged their properties (wavenumber, modeshapes)
- Both have a global motion of compressional type

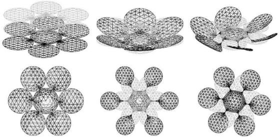


Figure: From left to right: real part (top) and imaginary part (bottom) of the displacement modeshape of branch 1 computed at points 1a, 1b and 1c.

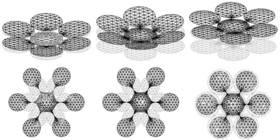
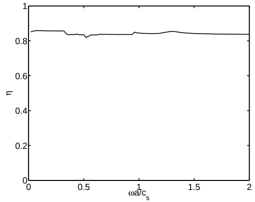
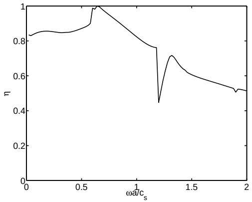


Figure: Modeshape of branch 2 computed at points 2a, 2b and 2c.

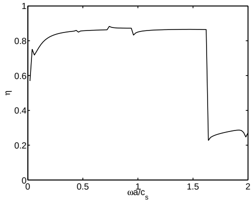
Let's change the excitation...



Excitation along z of a **peripheral** wire



Flexural excitation of central wire (x direction)



Torsional excitation of central wire
$\rightarrow$ existence of a $T'(0,1)$ mode