Simulations of multiple scattering by tree trunks using the TLM method

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   - TLM formulation for heterogeneous and dissipative media
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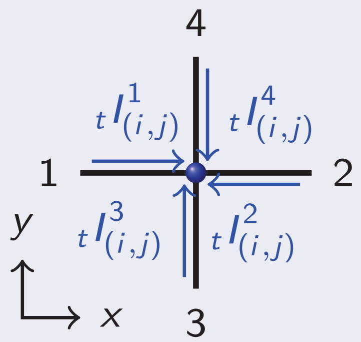
1/. The Transmission Line Matrix (TLM) method

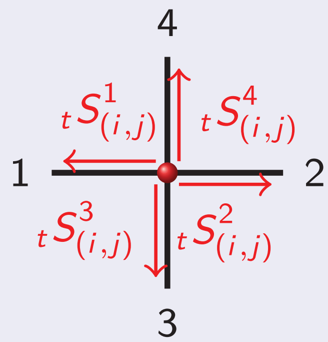
General principle of the TLM method

Huygens principle

Every point of a wave front may be considered the source of secondary wavelets that spread out in all directions with a speed equal to the speed of propagation of the waves.

TLM variables for 2D homogeneous and non-dissipative media





- Incident and scattered pulses:

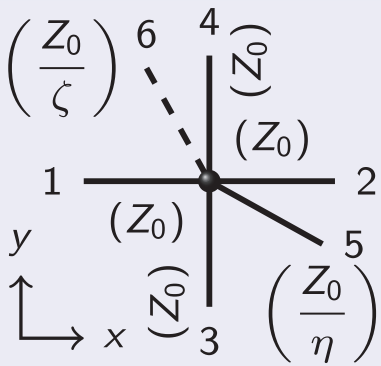
$$ {}_t\mathbf{I}_{(i,j)} = [\ {}_tI^1;\ {}_tI^2;\ {}_tI^3;\ {}_tI^4]^T_{(i,j)} $$

$$ {}_t\mathbf{S}_{(i,j)} = [\ {}_tS^1;\ {}_tS^2;\ {}_tS^3;\ {}_tS^4]^T_{(i,j)} $$

- Scattering process:

$$ {}_t\mathbf{S}_{(i,j)} = {}_t\mathbf{D}_{(i,j)}\ {}_t\mathbf{I}_{(i,j)} $$

TLM for heterogeneous and dissipative media



- Incident and scattered pulses:

$$ {}_t\mathbf{I}_{(i,j)} = [\ {}_tI^1;\ {}_tI^2;\ {}_tI^3;\ {}_tI^4;\ {}_tI^5]^T_{(i,j)} $$

$$ {}_t\mathbf{S}_{(i,j)} = [\ {}_tS^1;\ {}_tS^2;\ {}_tS^3;\ {}_tS^4;\ {}_tS^5]^T_{(i,j)} $$

- Scattering process:

$$ \mathbf{S} = \mathbf{D}(\eta, \zeta).\mathbf{I} $$

TLM formulation for heterogeneous and dissipative media

Connexion laws

$$
\begin{aligned}
{}_{t+\delta t}I^1_{(i,j)} &= {}_tS^2_{(i-1,j)},\\
{}_{t+\delta t}I^2_{(i,j)} &= {}_tS^1_{(i+1,j)},\\
{}_{t+\delta t}I^3_{(i,j)} &= {}_tS^4_{(i,j-1)},\\
{}_{t+\delta t}I^4_{(i,j)} &= {}_tS^3_{(i,j+1)},\\
{}_{t+\delta t}I^5_{(i,j)} &= {}_tS^5_{(i,j)}.
\end{aligned}
$$

Acoustic pressure

$$
{}_tp_{(i,j)} = \frac{2}{\eta_{(i,j)} + \zeta_{(i,j)} + 4}\left[\sum_{n=1}^{4} {}_tI^n_{(i,j)} + \eta_{(i,j)}\ {}_tI^5_{(i,j)}\right]
$$

TLM propagation scheme and wave equation

- TLM scheme for heterogeneous and dissipative network:

$$
\begin{aligned}
{}_{t+\delta t}p_{(i,j)} =& \frac{2}{\eta_{(i,j)} + \zeta_{(i,j)} + 4}\left[{}_tp_{(i+1,j)} + {}_tp_{(i-1,j)} + {}_tp_{(i,j+1)} + {}_tp_{(i,j-1)} + \eta_{(i,j)}\ {}_tp_{(i,j)}\right]\\
&- \frac{\eta_{(i,j)} - \zeta_{(i,j)} + 4}{\eta_{(i,j)} + \zeta_{(i,j)} + 4}\ {}_{t-\delta t}p_{(i,j)}
\end{aligned}
$$

- Wave equation:

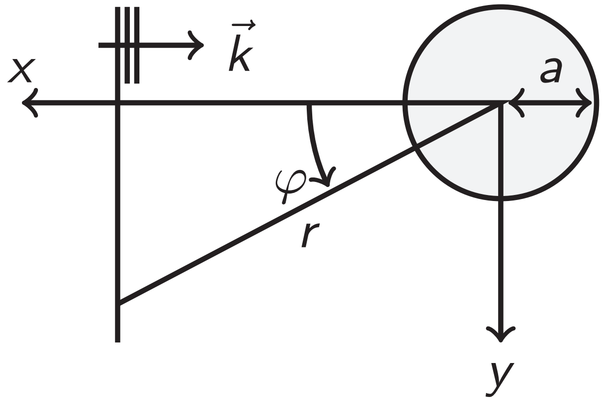
$$
\left[\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) - \frac{\eta+4}{2}\frac{\delta t^2}{\delta l^2}\frac{\partial^2}{\partial t^2} - \zeta\frac{\delta t}{\delta l^2}\frac{\partial}{\partial t}\right]\ {}_tp_{(i,j)} = 0
$$

- Celerity correction:

$$
c_{\text{TLM}} = \sqrt{\frac{2}{\eta+4}}\,c,\ \text{with } c = \frac{\delta l}{\delta t}
$$

2/. Scattering of a plane wave by a single cylinder

Diffraction of a plane wave by a circular scatterer



Schematic of wave propagation in the vicinity of a cylinder.

Analytical solutions [1]:

Incident plane wave: $p_i = P_0 \sum_{m=0}^{\infty} (2 - \delta_{m0}) i^m \; J_m \; (kr) \; e^{im\varphi} \; e^{i\omega t}$,

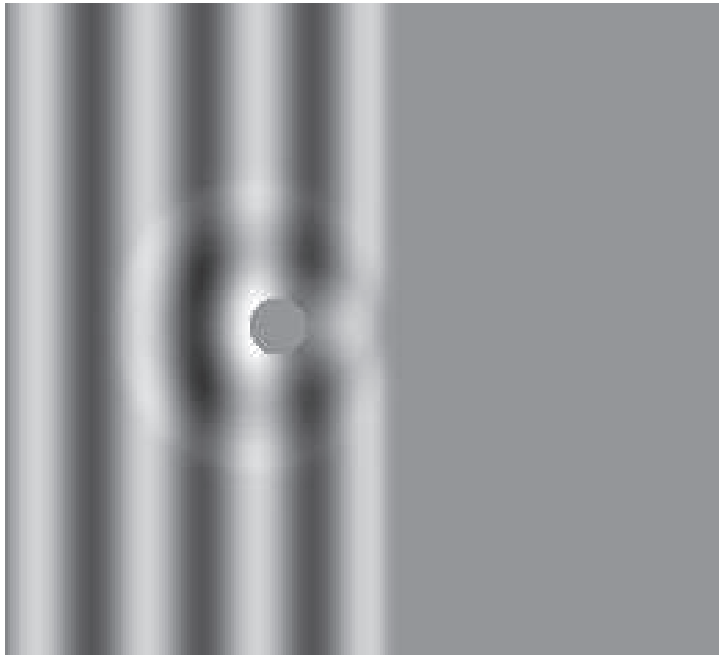
Scattered pressure: $p_s = P_0 \sum_{n=0}^{\infty} A_n H_n^{(1)} \; (kr) \; e^{in\varphi} \; e^{i\omega t}$,

Boundary condition at $r = a$: $\frac{i}{k\rho c} \frac{\partial}{\partial r}(p_i + p_s) = \frac{-1}{Z}(p_i + p_s)$,

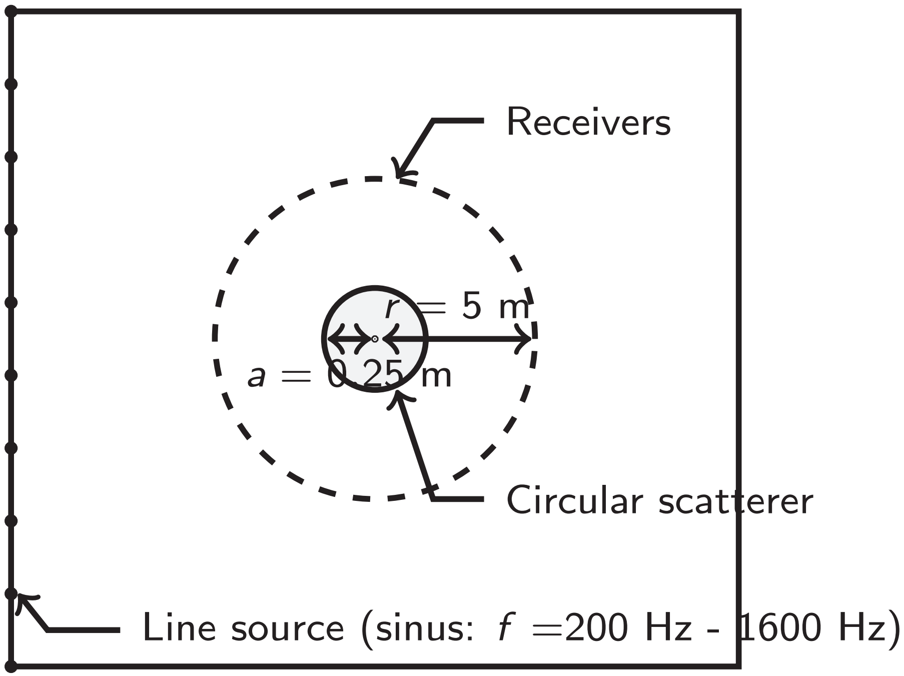
Scattering coefficients: $A_n = -\frac{(2-\delta_{n0}) i^n [i J_n'(ka) + (\rho c / Z) J_n(ka)]}{i H_n^{(1)'}(ka) + (\rho c / Z) H_n^{(1)}(ka)}$.

[1] Bruneau, M.; Hermès (Ed.) Manuel d'acoustique fondamentale, Hermès (1998).

Principle and geometries



TLM simulation of plane wave scattered by a circular scatterer.

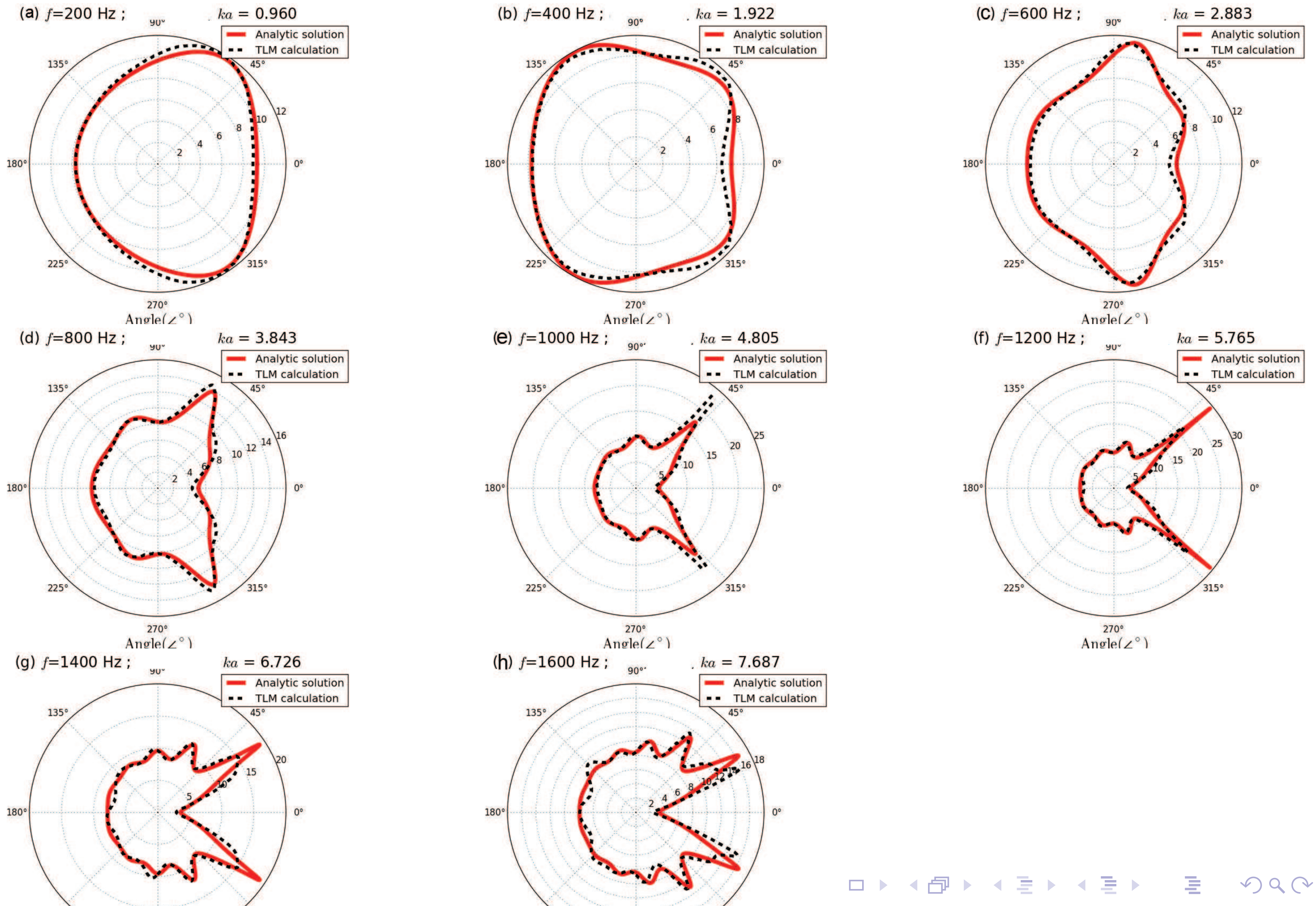


Simulations' geometrical setup.

Scattered level pressure

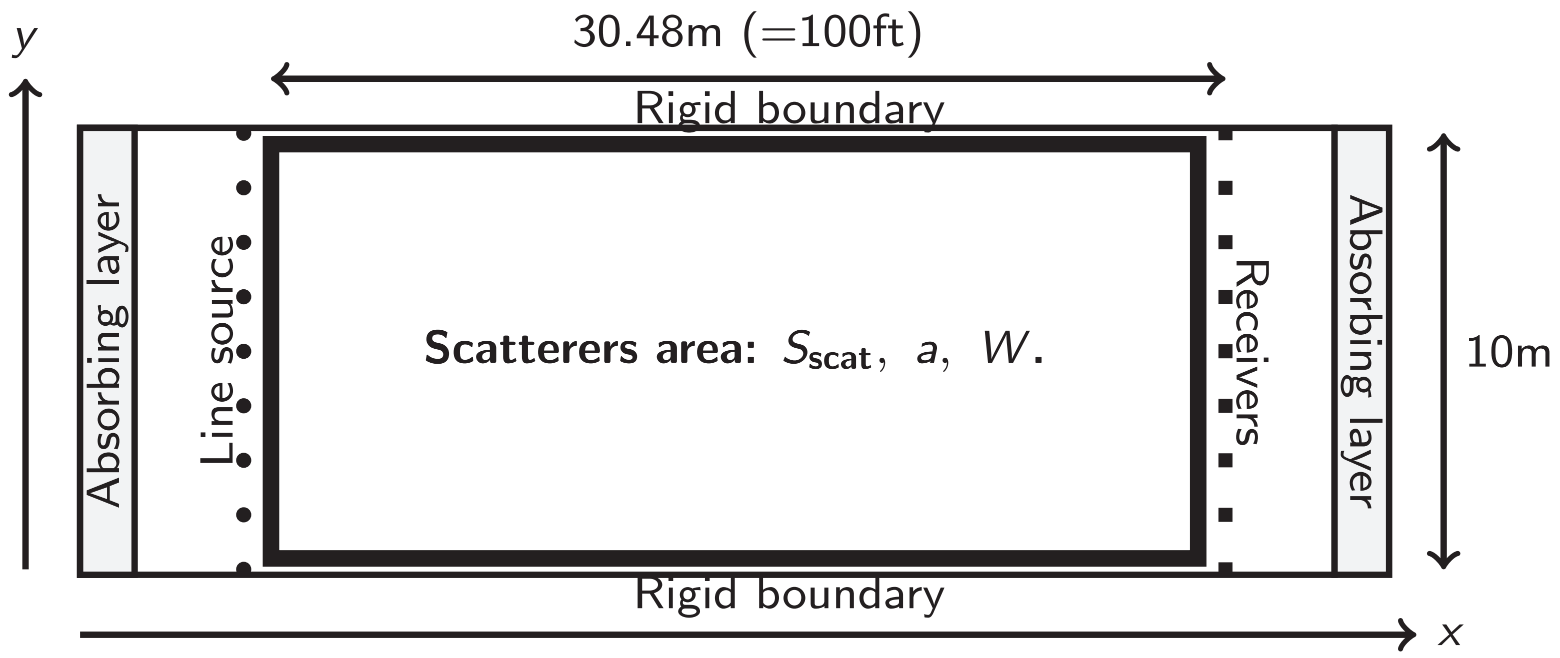
$$L_{\text{scat}} = 10 \log_{10} \left(\frac{p_{s,\text{rms}}}{p_{i,\text{rms}}} \right) .$$

Comparison between analytical solutions and the TLM results



3/. Multiple scattering

Geometry of the simulations



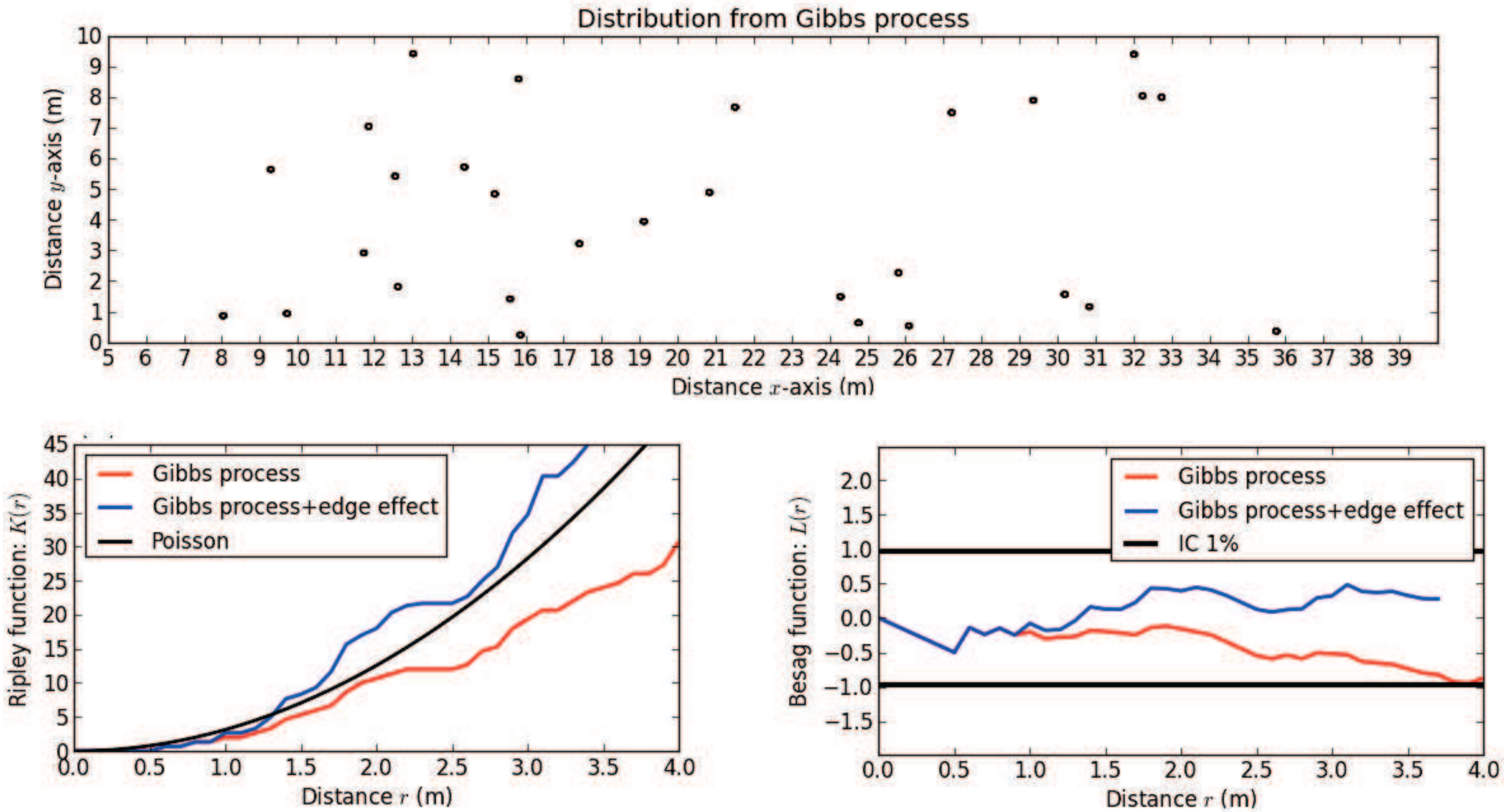
Quantity of interest:

The mean pressure density referenced to an 100 feet long scatterers array [4]:

$$EL = 10 \log_{10} \left(\frac{|\psi_t|^2}{|\psi_{t,100ft}|^2} \right).$$

[4] Embleton, T.; Scattering by an array of cylinders as a function of surface impedance, *J. Acoust. Soc. Am.*, **40**, 667-670 (1966).

Distribution process for the scatterers locations



Assessment of the scatterers locations:

Ripley's funct.[2]: $K(r) = \frac{1}{W}\frac{1}{N}\sum_{i=1}^{N}\sum_{j\neq i} k_{ij}$, Besag's funct.[3]: $L(r) = \sqrt{\frac{K(r)}{\pi}} - r$.

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[2] Ripley B.; The second order analysis of stationary point process, *Journal of applied probability*, **13**, 255-266 (1976).

[3] Besag J.; Contribution to the discussion of Dr Ripley's paper, *Journal of the royal statistical society*, **B 39**, 193-195 (1977).

Theory from Twersky's average wave-functions

Variables of interest [4,5]:

Average wave transmitted: $T = \frac{2W}{k} \sum_{-\infty}^{\infty} A_n,$

Average wave reflected: $R = \frac{2N}{k} \sum_{-\infty}^{\infty} (-1)^n A_n \ ,$

Scattering coefficients: $A_n = -\frac{iJ_n(kr)+(Z/\rho c)J_n'(kr)}{iH_n(kr)+(Z/\rho c)H_n'(kr)},$

Complex propagation constant: $\gamma = \sqrt{(k - iT)^2 + R^2},$

Balance between transmitted and back-scattered field : $q = (T + ik - i\gamma)/R \ ,$

Internal wave field: $\psi_i = (1-q)\frac{e^{i\gamma x}+qe^{-i\gamma(x-2d)}}{1-q^2 e^{i2\gamma d}},$

Transmitted wave field: $\psi_t = (1-q^2)\frac{e^{i(\gamma-k)d}}{1-q^2 e^{i2\gamma d}}.$

[4] Embleton, T.; Scattering by an array of cylinders as a function of surface impedance, *J. Acoust. Soc. Am.*, **40**, 667-670 (1966).

[5] Twersky, V.; On scattering of waves by random distributions. I. Free space scatterer formalism, *J. Math. Phys.*,**3**,700-715(1962).

Comparison between analytical solutions and the TLM results

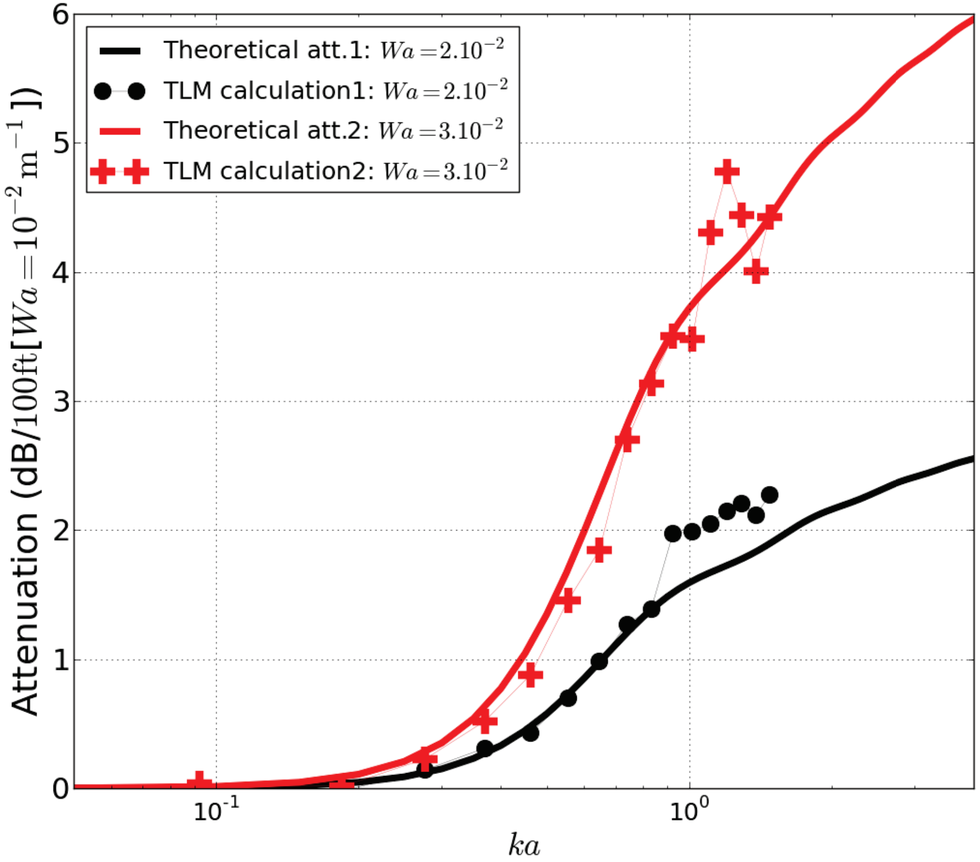


Figure: Attenuations calculated with a reference domain where $Wa = 10^{-2}\text{m}^{-1}$ for a scatterers domain where $Wa = 2.10^{-2}\text{m}^{-1}$ (black points), and a scatterers domain where $Wa = 3.10^{-2}\text{m}^{-1}$ (red crosses) in comparison to the corresponding theoretical attenuations.

4/. Conclusion and future work

Conclusion

Validations of the TLM method for 2D scattering cases

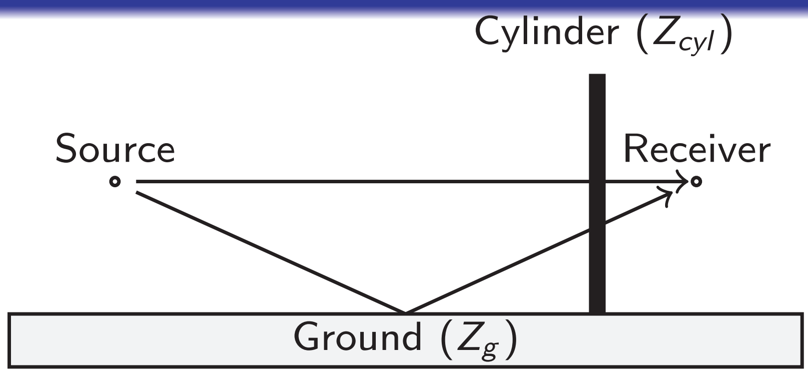
- scattering of a plane wave by a perfectly rigid circular scatterer,
- multiple scattering by randomly located circular scatterers:
  → scatterers locations related to forests' spatial structure analysis.

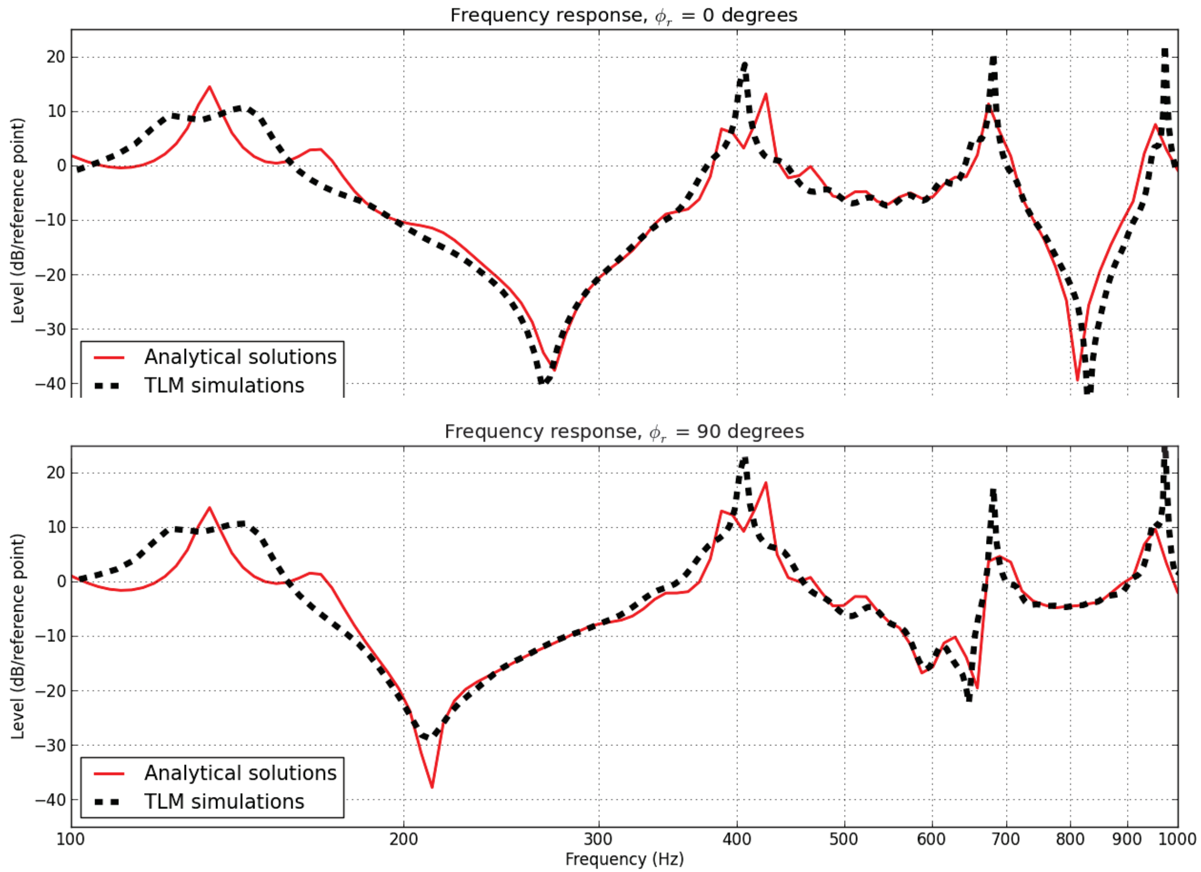
Future work on 3D cases with impedance plane and cylinders

- single scattering of a cylinder placed normal to an impedance plane:
  - analytical analysis from M. Swearingen semi-analytical model [6],
  - measurements on scale models with reflecting and felt-covered ground.
- multiple scattering from cylinders placed normal to an impedance plane:
  - measurements on scale models for six configurations:
    → three different placements (periodical aligned, periodical in staggered-row, random),
    → two different ground impedances (rigid and felt-covered grounds).

[6] Swearingen M., Swanson D.; A numerical model for point source scattering from an impedance cylinder placed normal to an impedance ground, *Acta Acustica*, **98**, 523-533 (2012).

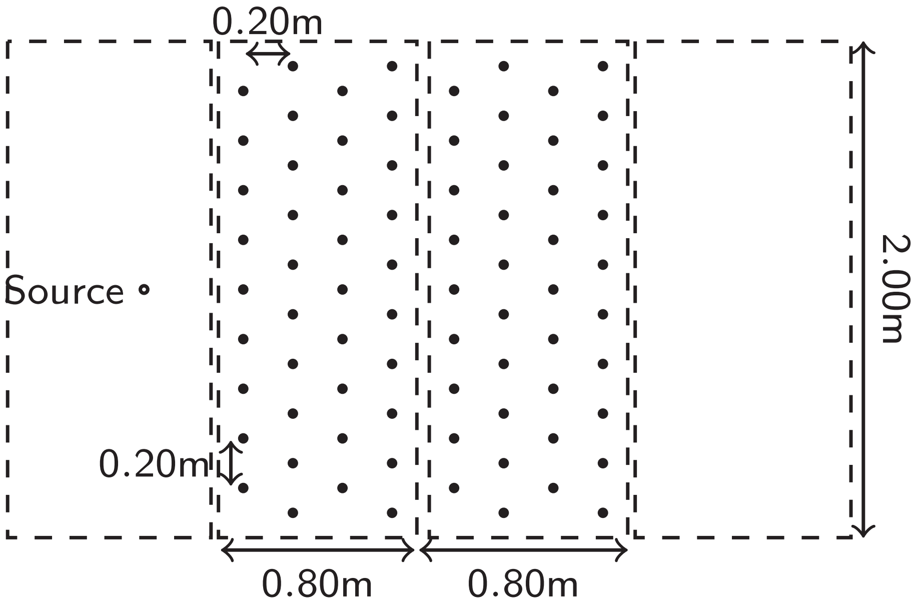
TLM and analytical results for a cylinder placed normal to a plane



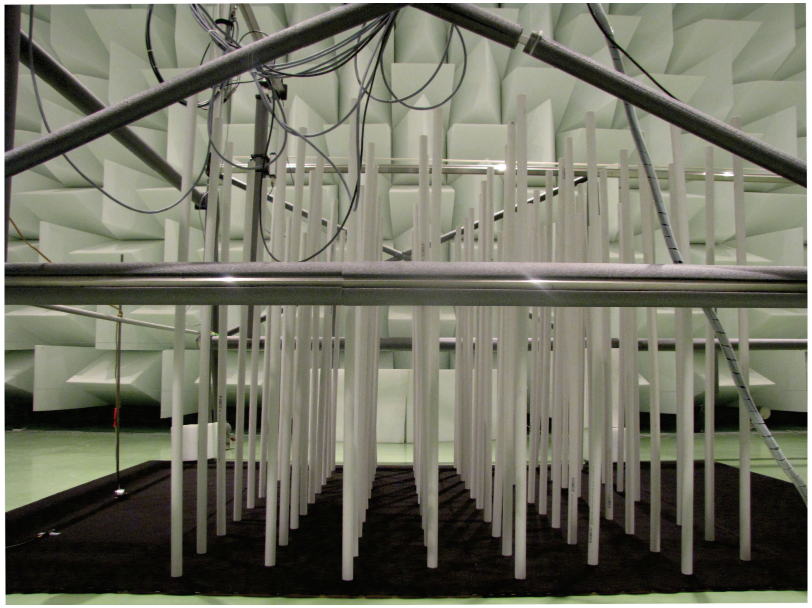


[6] Swearingen M., Swanson D.; A numerical model for point source scattering from an impedance cylinder placed normal to an impedance ground, *Acta Acustica*, **98**, 523-533 (2012).

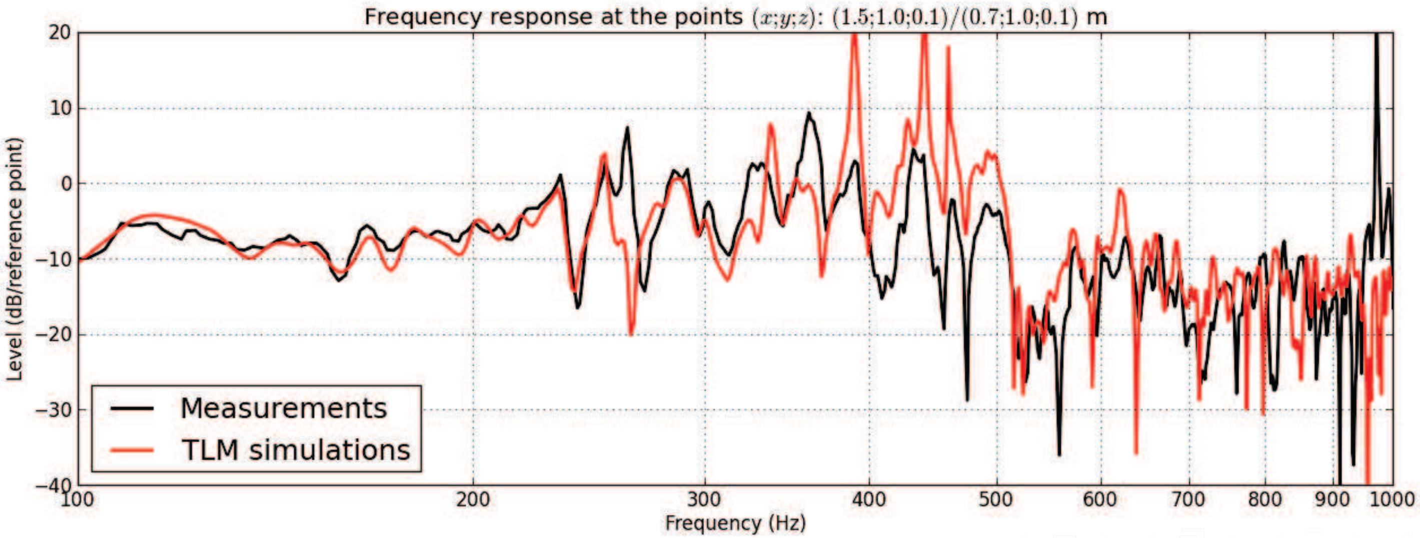
Measurements on scale models for multiple scattering



Top view of the scale model for a periodical distribution of the cylinders.



Lateral view of the experimental setup.



Thanks for your attention

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