Optimisation d'admittance appliquée à la conception d'une barrière antibruit de faible hauteur

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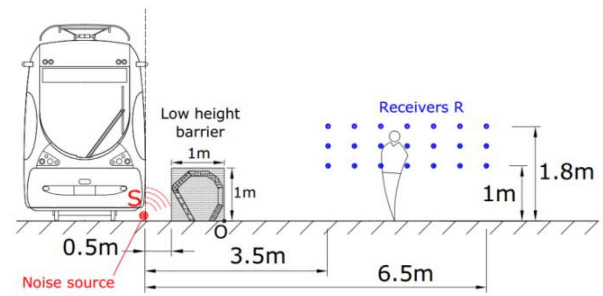
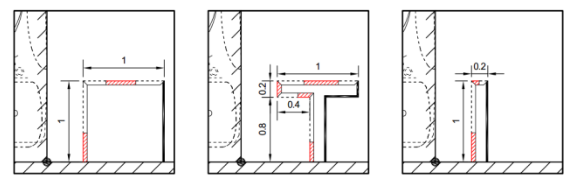
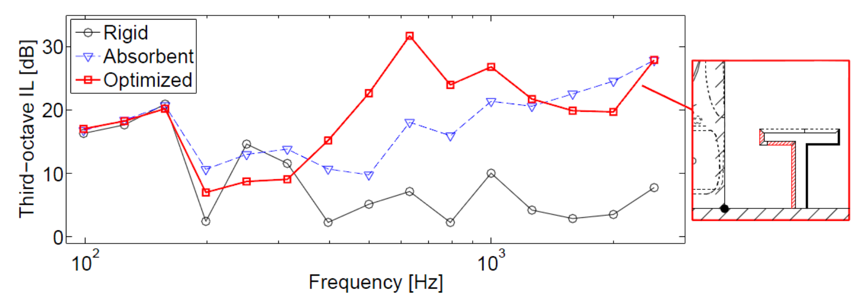






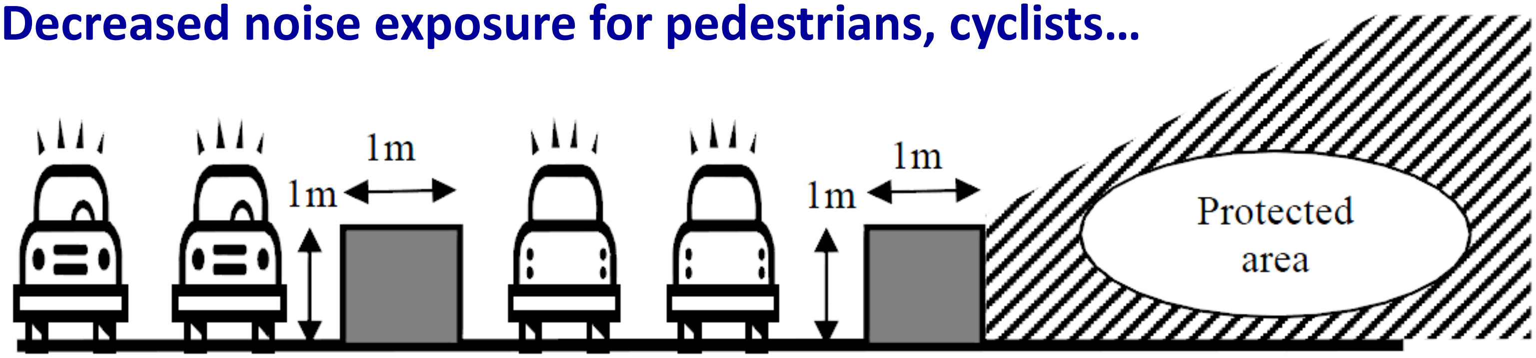
Journées Techniques “Acoustique et Vibrations”

Autun – le 10 et 11 octobre 2012

Low-height noise barriers can be an efficient way to create quiet zones close to transportation routes in urban areas

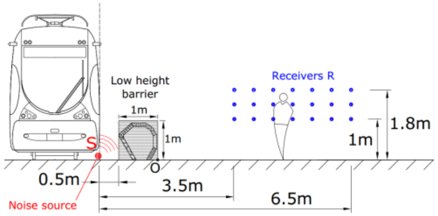
kworth30, Flickr

Decreased noise exposure for pedestrians, cyclists…



Source : • M. Baulac, « Optimisation des protections antibruit routières de forme complexe », thèse de doctorat, Université du Maine (Le Mans, France), 2006
• F. Koussa, « Évaluation de la performance acoustique des protections antibruit innovantes utilisant des moyens naturels : application aux transports terrestres », thèse de doctorat, Ecole Centrale de Lyon, 2012

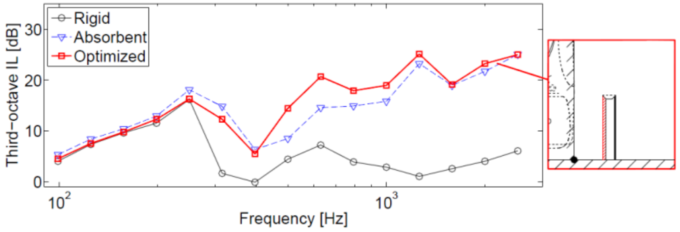
This talk introduces a gradient-based optimization method to design the surface treatment of a low-height barrier



Implementation of the barrier

$$\frac{\mathrm{d}P}{\mathrm{d}\beta}(\beta, p_\Gamma) = \frac{\partial \mathcal{L}}{\partial \beta}(\beta, p_\Gamma, q_\Gamma)$$

Gradient calculation



Optimization results

* A. Jolibois, D. Duhamel, V.W. Sparrow, J. Defrance, P. Jean, "Application of admittance optimization to the design of a low-height tramway noise barrier", Proceedings of Internoise 2012 in New York City (August 2012)

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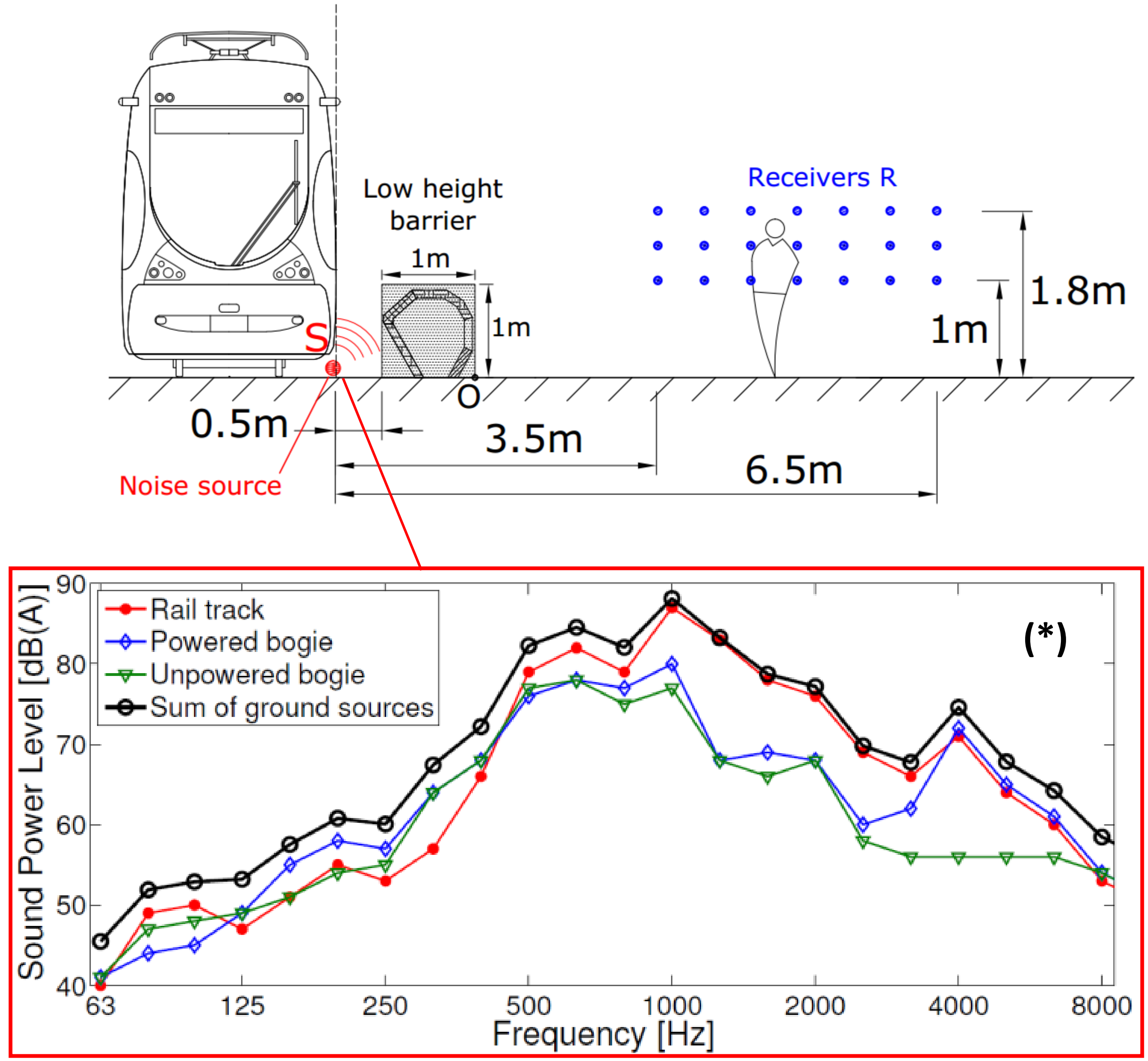
A low-height noise barrier close to a tramway has been considered

Source
Tramway noise
Line source on ground

Barrier
Arbitrary fixed geometry
Holds in a 1m wide square
Arbitrary admittance

Physical conditions
Homogeneous atmosphere
Infinitely long barrier (2D approx.)
Locally reacting surface treatment
Rigid ground
Reflection on tramway side : baffle



***Source:** M. A. Pallas, J. Lelong, R. Chatagnon, “Characterization of tram noise emission and contribution of the noise sources”, Appl. Acoust. **72**, 437-450 (2011)

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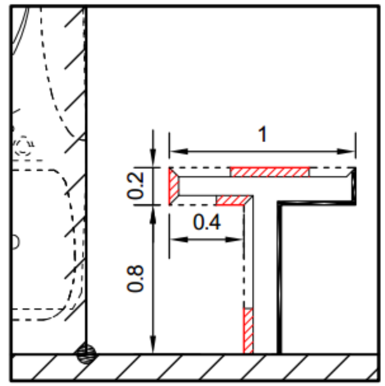
A T-shape geometry and two kinds of admittances have been considered for the barrier coverage

Porous

MPP

Rigid



T-shape

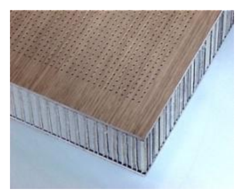
Porous (thick grass): Delany & Bazley layer[1]

$$\begin{cases} \tilde{z}(f) = 1 + 0.0511\left(\frac{\sigma}{f}\right)^{0.75} + \mathrm{i}\,0.0768\left(\frac{\sigma}{f}\right)^{0.73} \\ \frac{\tilde{k}}{k_0} = 1 + 0.0858\left(\frac{\sigma}{f}\right)^{0.7} + \mathrm{i}\,0.175\left(\frac{\sigma}{f}\right)^{0.59} \end{cases}$$

$$z_{\mathrm{DB}}(f) = \tilde{z}(f)\coth(-\mathrm{i}\tilde{k}d)$$

Flow resistivity [kPa s/m^2]: $\sigma_{min} = 50$; $\sigma_{max} = 200$ ([2])
Layer thickness [cm]: $d_{min} = 1$; $d_{max} = 10$

Micro-perforated panel (MPP)[3,4]



$$z_{\mathrm{MPP}}(f) = -\mathrm{i}\,\frac{kl_0}{s}\left(\frac{1}{\Theta(x')} + \frac{16}{3\pi}\,\frac{a_0}{l_0}\,\frac{\Psi(\xi)}{\Theta(x)}\right) + \mathrm{i}\,\mathrm{cotan}\,(kD)$$

$$k = \frac{2\pi f}{c_0}\ ;\ \xi = \sqrt{s}\ ;\ x = a_0\sqrt{\frac{2\pi f \rho_0}{\mu}}\ ;\ x' = a_0\sqrt{\frac{2\pi f \rho_0}{\mu'}}$$

Porosity: $s_{min} = 0.01$; $s_{max} = 0.4$
Hole radius [mm]: $a_{0,min} = 0.5$; $a_{0,max} = 5$
Panel thickness [cm]: $l_{0,min} = 0.2$; $l_{0,max} = 1$
Cavity depth [cm]: $D_{min} = 1$; $D_{max} = 10$

Source:
[1] Delany and Bazley, "Acoustical properties of fibrous absorbent materials", Appl. Acoust. **3**, 105-116 (1970)
[2] Attenborough et al., "Outdoor ground impedance models", J. Acoust. Soc. Am. **129**(5), 2806-2819 (2011)
[3] Maa, "Potential of microperforated panel absorber", J. Acoust. Soc. Am. **104**(5), 2861-2866 (1998)
[4] Asdrubali and Pispola, "Properties of transparent sound-absorbing panels for use in noise barriers", J. Acoust. Soc. Am. **21**(1), 214-221 (2007)

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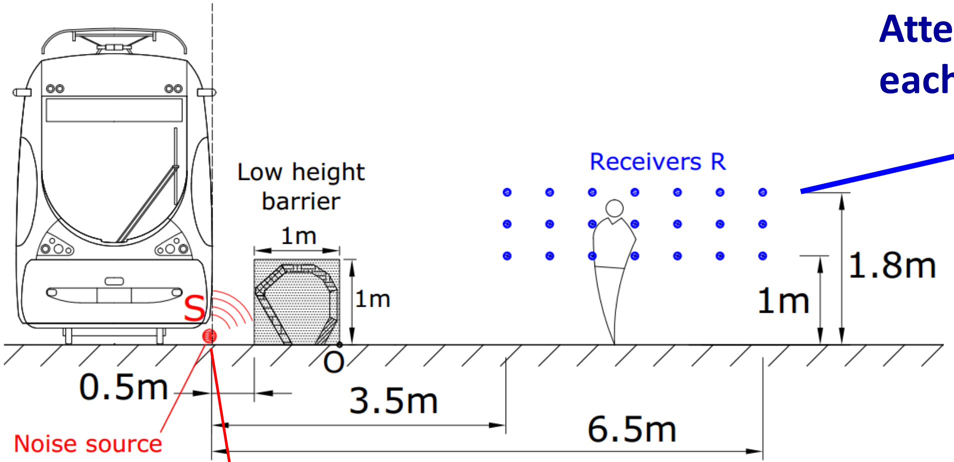
This goal is to minimize the noise reaching the receiver zone by designing the admittance distribution



Attenuation at each frequency

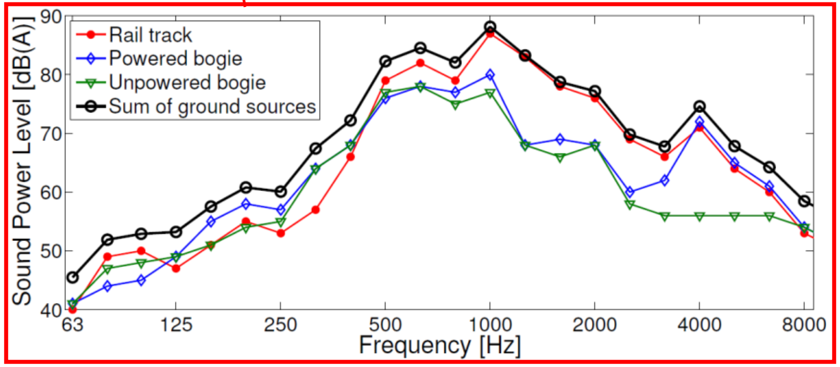
$$A_n = \left[\frac{\sum_i |p(R_i, f_n)|^2}{\sum_i |p^{\text{in}}(R_i, f_n)|^2}\right]^{\frac{1}{2}} = \frac{P(f_n)}{P^{\text{in}}(f_n)}$$

Weighted attenuation

$$e = \frac{\sum_n S_n A_n^2}{\sum_n S_n}$$

Broadband insertion loss

$$\text{IL} = -10 \log e$$



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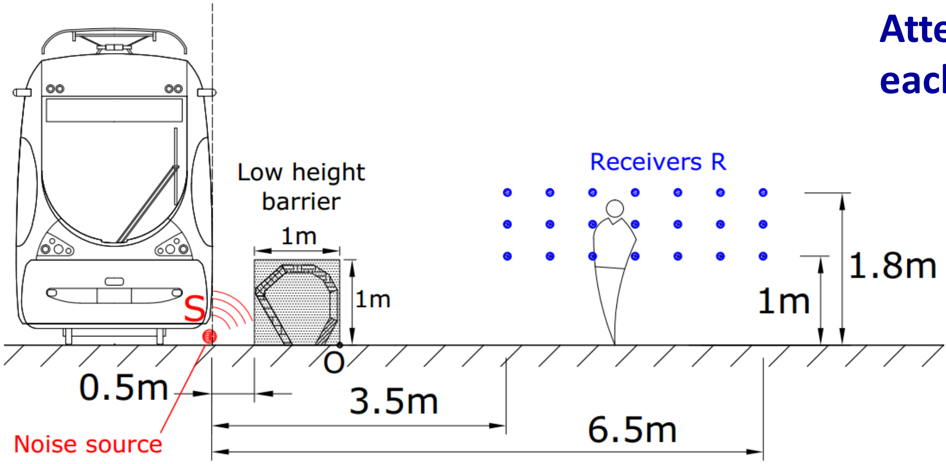

Attenuation at each frequency

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Weighted attenuation

$$e = \frac{\sum_n S_n A_n^2}{\sum_n S_n}$$

Broadband insertion loss

$$\text{IL} = -10 \log e$$

Minimize *e* (maximize IL)

- Gradient-based algorithm : SQP
- f_n : 6 frequencies per octave (100-2500 Hz)
- 5 random starting points

Optimization → **Panel parameters**, **Width of each panel**

Need the gradient $\dfrac{\mathrm{d}e}{\mathrm{d}a} \rightarrow \dfrac{\mathrm{d}A_n}{\mathrm{d}a} \rightarrow \boxed{\dfrac{\mathrm{d}P(f_n)}{\mathrm{d}a}}$

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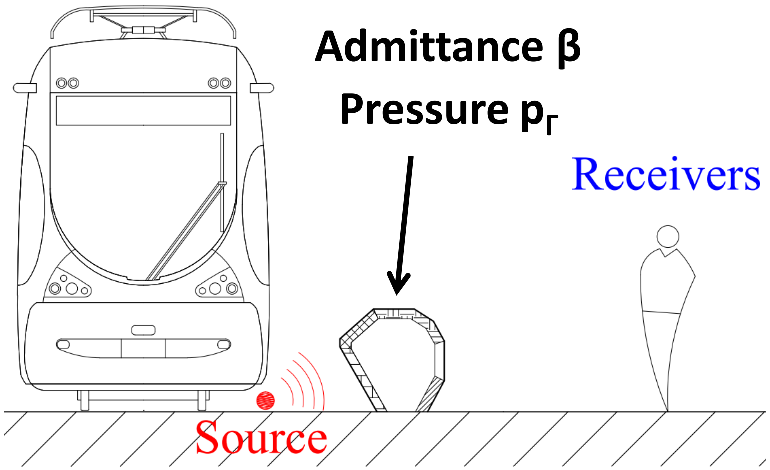
The gradient w.r.t to the admittance parameters is calculated efficiently using the BEM and the adjoint state approach



$$P = \left[\sum_i |p(R_i)|^2\right]^{\frac{1}{2}} \xrightarrow{\text{BEM}} P = P(\beta, p_\Gamma)$$

State: $p_\Gamma = p_\Gamma(\beta)$

(Primal BEM integral equation)

$$P = P(\beta, p_\Gamma(\beta)) \quad \Rightarrow \quad \frac{\mathrm{d}P}{\mathrm{d}\beta} = \frac{\partial P}{\partial \beta} + \frac{\partial P}{\partial p_\Gamma} \circ \frac{\mathrm{d}p_\Gamma}{\mathrm{d}\beta}$$

Implicit function

Using the adjoint state* : q_Γ **(Dual BEM integral equation)**

$$\frac{\mathrm{d}P}{\mathrm{d}\beta}(\beta, p_\Gamma) = \frac{\partial \mathcal{L}}{\partial \beta}(\beta, p_\Gamma, q_\Gamma) \longrightarrow$$

Explicit function

*__Source:__ G. Allaire, ”Conception optimale des structures”, Springer (2007)

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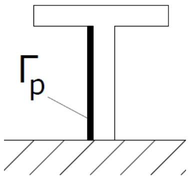
A very simple expression for the gradient with respect to the admittance can then be written

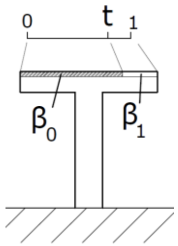
$$\boxed{\frac{\mathrm{d}P}{\mathrm{d}\beta}(\beta) = \frac{\partial \mathcal{L}}{\partial \beta}(\beta, p_\Gamma, q_\Gamma) = ik\, p_\Gamma\, q_\Gamma}$$

<u>Gradient with respect to a model parameter</u>

$$\frac{\mathrm{d}P}{\mathrm{d}\sigma} = \mathrm{Re}\left[ik\, \frac{\mathrm{d}\beta_p}{\mathrm{d}\sigma} \int_{\Gamma_p} p_\Gamma\, q_\Gamma \right]$$



β_p : admittance on Γ_p



<u>Gradient with respect to a panel width</u>

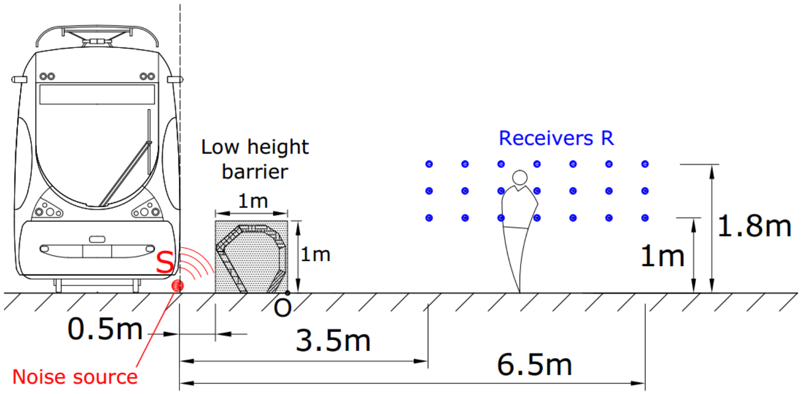
$$\frac{\mathrm{d}P}{\mathrm{d}t} = \mathrm{Re}\left(ik\, (\beta_0 - \beta_1)\, p_\Gamma(t)\, q_\Gamma(t)\, J_\Gamma(t) \right)$$

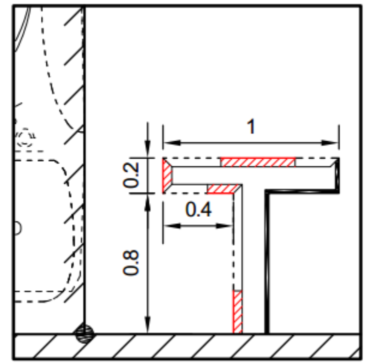
<u>Computational aspects</u>

State and adjoint state calculation ⟶ 2 BEM problems ⟶ x2 CPU time

Gradient calculation ⟶ Explicit integral ⟶ Negligible CPU time

Results show good improvement of the barrier performance especially in the mid-frequency range

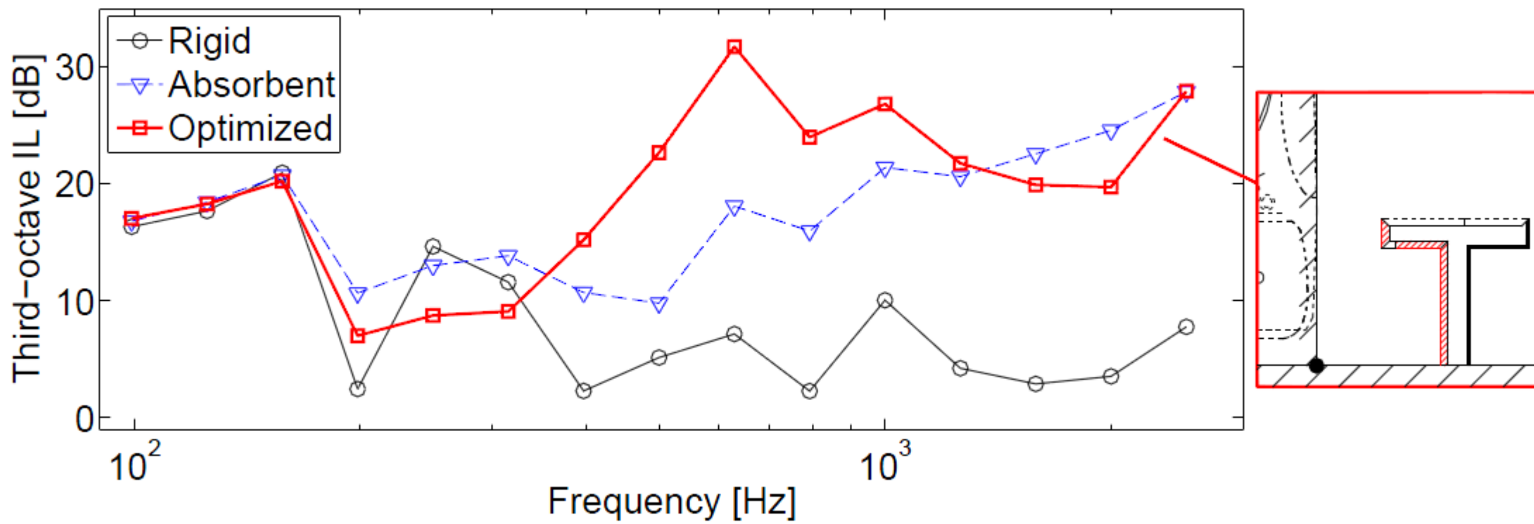




T-shape

Rigid	5.6 dB(A)
Absorbent	16.6 dB(A)
Optimized	23.1 dB(A)

Optimization gain: 6.5 dB(A)



Absorbent: D&B layer – σ = 50 kPa s/m², d = 10 cm

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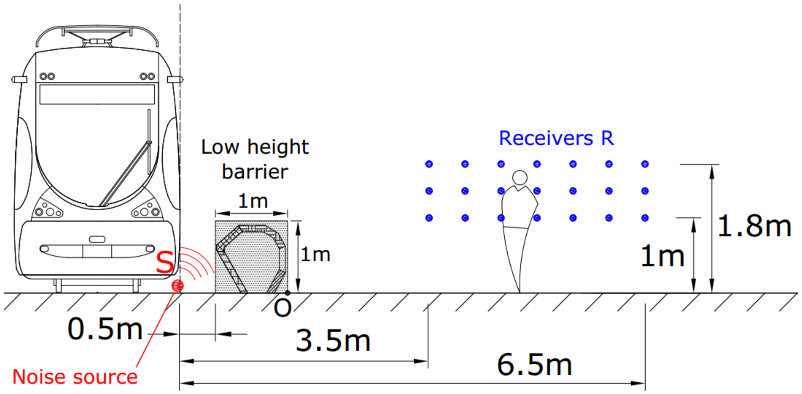

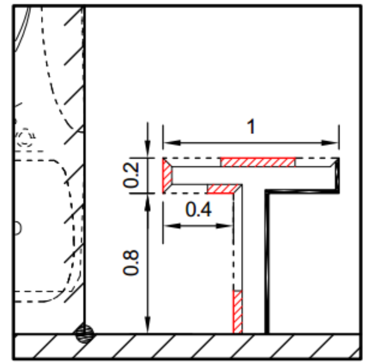

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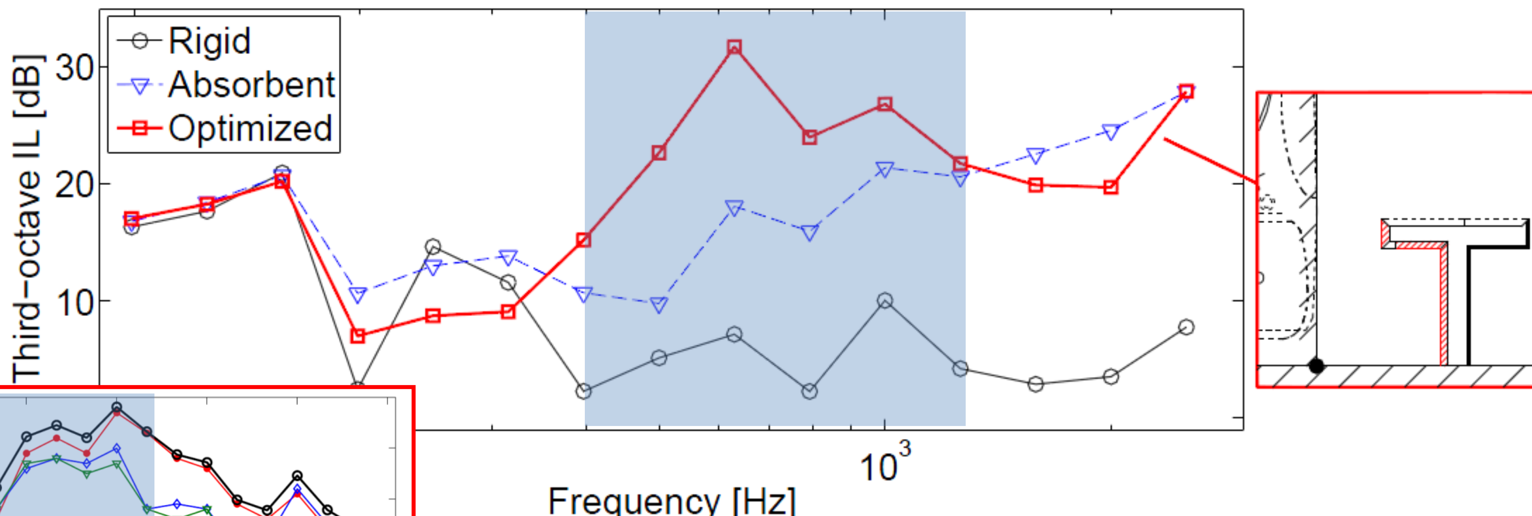

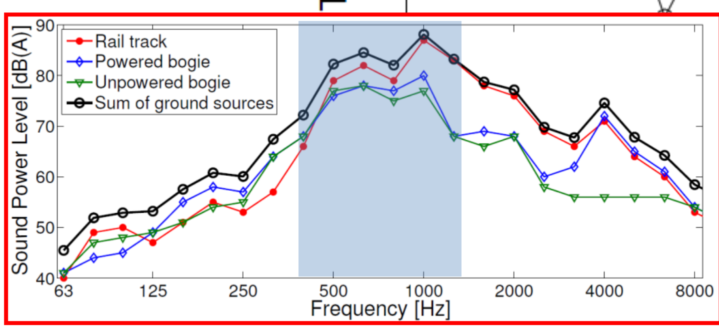

Absorbent: D&B layer – σ = 50 kPa s/m², d = 10 cm

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In summary, admittance optimization allows to design low-height noise barriers surface treatment efficiently

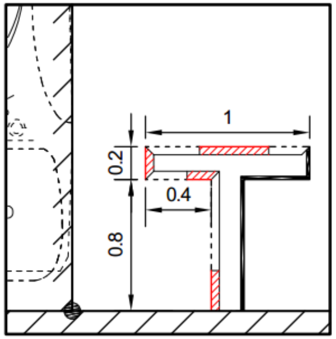
$$\frac{\mathrm{d}P}{\mathrm{d}\beta}(\beta) = \frac{\partial \mathcal{L}}{\partial \beta}(\beta, p_\Gamma, q_\Gamma) = ik\, p_\Gamma\, q_\Gamma$$

Adjoint state

- ⟶ **Sensitivity (gradient) w.r.t. admittance parameters**
- ⟶ **Negligible extra CPU time**

Gradient-based optimization algorithm

- ⟶ **Ex: MPP and porous layers**
- ⟶ **Optimization gain of 6 dB(A)**

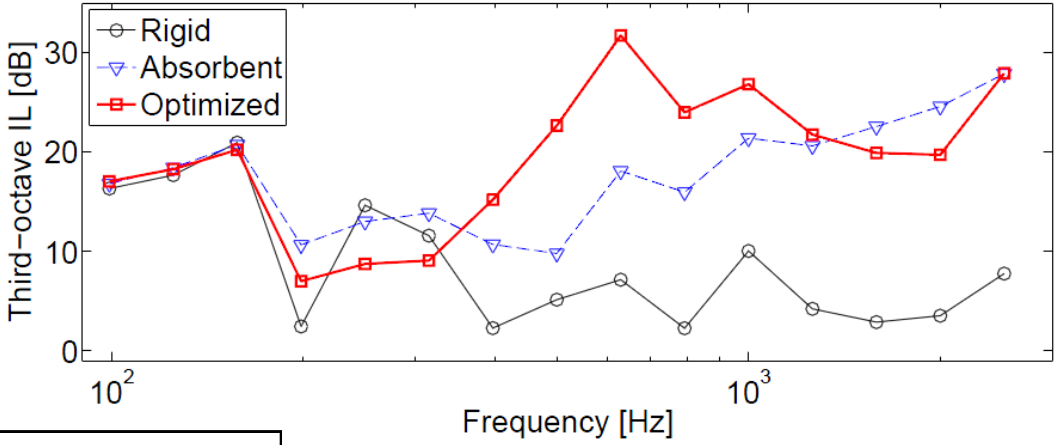


Porous materials

- ⟶ **Baseline absorption**
- ⟶ **Prevent reverberant build-up**

MPP

- ⟶ **Automatic tuning with optimization**
- ⟶ **Further increase attenuation at mid-frequencies**



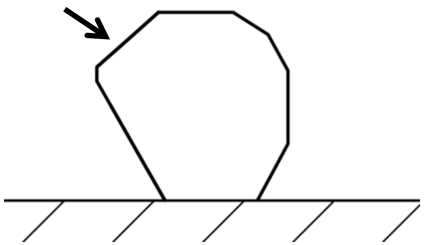
Questions ?

BACKUP SLIDES

The gradients involved are complex functional gradients defined on the barrier

Notation $\langle a, b \rangle = \int_\Gamma ab \, \mathrm{d}\Gamma$

Curve Γ



D **: set of smooth complex functions defined on Γ**
F functional on D
Linear approximation of F about f:

$$(\forall g \in D) \quad F(f+g) = F(f) + L_f(g) + o\big(\, ||g||_\infty \,\big)$$

L_f : linear form on *D* (differential)

⟶ **Identification with a complex function** $L_f(g) = \left\langle \frac{\mathrm{d}F}{\mathrm{d}f}, g \right\rangle$ **"Gradient" of F**

If F is real $$L_f(g) = \mathrm{Re} \left\langle \frac{\mathrm{d}F}{\mathrm{d}f}, g \right\rangle$$

Properties $$\frac{\mathrm{d}|F|^2}{\mathrm{d}f} = 2\, F^* \frac{\mathrm{d}F}{\mathrm{d}f} \quad \text{and} \quad \frac{\mathrm{d}|F|}{\mathrm{d}f} = \frac{F^*}{|F|} \frac{\mathrm{d}F}{\mathrm{d}f}$$

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The sound field resolution comes down to the determination of the pressure on the scatterer (the state)

G: Green's function ⟵ **(Rigid) ground reflection + coherent line source (2D)**

$$G(x_1,x_2,y_1,y_2) = \frac{\mathrm{i}}{4}\left(H_0\left[k\sqrt{(y_1-x_1)^2+(y_2-x_2)^2}\right] + H_0\left[k\sqrt{(y_1-x_1)^2+(y_2+x_2)^2}\right]\right)$$

Given a pressure distribution p on Γ, define:

$$Sp \;:\; x \mapsto \int_\Gamma G(\mathbf{x},\mathbf{y})\,p(\mathbf{y})\,\mathrm{d}\Gamma(\mathbf{y})$$

$$Dp \;:\; x \mapsto \int_\Gamma \frac{\partial G}{\partial n_2}(\mathbf{x},\mathbf{y})\,p(\mathbf{y})\,\mathrm{d}\Gamma(\mathbf{y})$$

$$D^*p \;:\; x \mapsto \int_\Gamma \frac{\partial G}{\partial n_1}(\mathbf{x},\mathbf{y})\,p(\mathbf{y})\,\mathrm{d}\Gamma(\mathbf{y})$$

$$Np \;:\; x \mapsto \int_\Gamma \frac{\partial^2 G}{\partial n_1\,\partial n_2}(\mathbf{x},\mathbf{y})\,p(\mathbf{y})\,\mathrm{d}\Gamma(\mathbf{y})$$

Adjoint properties

$$\langle Sp,q\rangle = \langle Sq,p\rangle$$

$$\langle Dp,q\rangle = \langle D^*q,p\rangle$$

$$\langle Np,q\rangle = \langle Nq,p\rangle$$

Incident and scattered field

$$p = p^{\mathrm{in}} + p^{\mathrm{sc}}$$

$$p^{\mathrm{in}}(\mathbf{x}) = G(S,\mathbf{x})$$

Scattering problem

$$\begin{cases} -(\nabla^2+k^2)\,p^{\mathrm{sc}} = 0 \quad \text{in } \Omega^e \\ \dfrac{\partial p^{\mathrm{sc}}}{\partial n} + \mathrm{i}k\,\beta\,p^{\mathrm{sc}} = h_1^{\mathrm{in}}(\beta) \quad \text{on } \Gamma \qquad \text{with} \quad h_1^{\mathrm{in}}(\beta) = -\left.\dfrac{\partial p^{\mathrm{in}}}{\partial n}\right|_\Gamma - \mathrm{i}k\beta\,p^{\mathrm{in}}|_\Gamma \\ + \quad \text{radiation condition} \end{cases}$$

State

Integral representation

(Kirchhoff-Helmhotz integral theorem)

$$p^{\mathrm{sc}}(R_i) = \int_\Gamma \left(\frac{\partial G}{\partial n_2}(R_i,\mathbf{y}) + ik\beta(\mathbf{y})\,G(R_i,\mathbf{y})\right) p_\Gamma(\mathbf{y})\,\mathrm{d}\mathbf{y}$$

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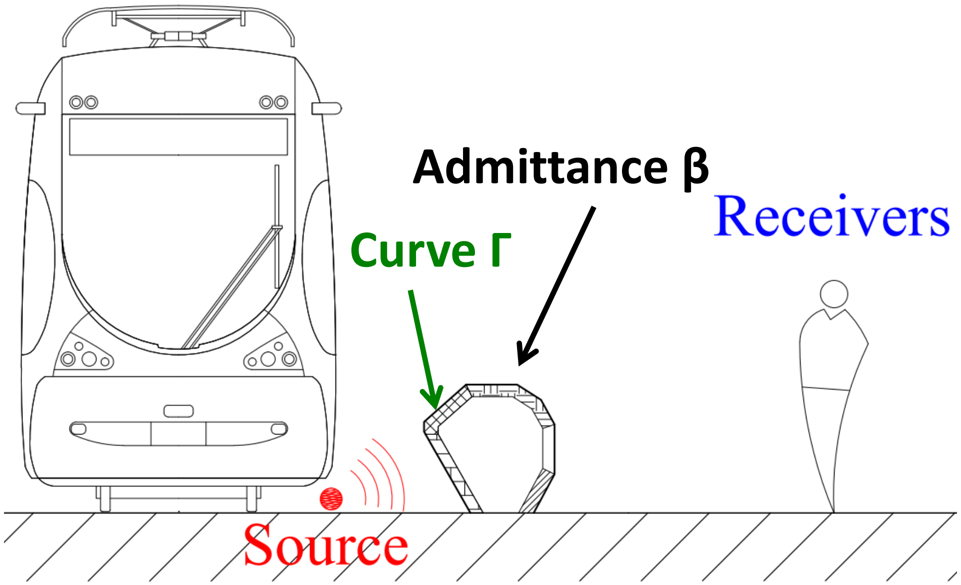
The objective function depends on the admittance and the shape both explicitly and implicitly



Integral representation (BEM)

$$P = \left[\sum_i |p(R_i)|^2\right]^{\frac{1}{2}}$$

$$p(R_i) = p^{\text{in}}(R_i) + p^{\text{sc}}(R_i)$$

$$p^{\text{sc}}(R_i) = \int_\Gamma \left(\frac{\partial G}{\partial n_2}(R_i, \mathbf{y}) + ik\beta(\mathbf{y})\, G(R_i, \mathbf{y})\right) p_\Gamma(\mathbf{y})\, \mathrm{d}\mathbf{y}$$

$$\longrightarrow \quad \boxed{P = P(\beta, p_\Gamma)}$$

State p_Γ → State equation : <u>primal BEM problem (MICADO*)</u>

$$N p_\Gamma + D^*(\mathrm{i}k\beta p_\Gamma) + \mathrm{i}k\beta\, D p_\Gamma + \mathrm{i}k\beta\, S(\mathrm{i}k\beta p_\Gamma) = h_1^{\text{in}}(\beta)$$

$$h_1^{\text{in}}(\beta) = -\frac{\partial p^{\text{in}}}{\partial n}\bigg|_\Gamma - \mathrm{i}k\beta\, p^{\text{in}}|_\Gamma$$

→ **State p_Γ is an implicit function of admittance β**

$$P = P(\beta, p_\Gamma(\beta)) \quad \Rightarrow \quad \frac{\mathrm{d}P}{\mathrm{d}\beta} = \frac{\partial P}{\partial \beta} + \frac{\partial P}{\partial p_\Gamma} \circ \frac{\mathrm{d}p_\Gamma}{\mathrm{d}\beta}$$

Implicit function

* **<u>Source:</u>** P. Jean, "A variational approach for the study of outdoor sound propagation and application to railway noise," J. Sound Vib. **212** (2), 275-294 (1998).

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The adjoint state is introduced to avoid dealing with the implicit dependence of the state on the parameters

Define the Lagrangian

$$\boxed{\mathcal{L}(\hat{\beta}, \hat{p}_\Gamma, \hat{q}_\Gamma) = P(\hat{\beta}, \hat{p}_\Gamma) + Q(\hat{\beta}, \hat{p}_\Gamma, \hat{q}_\Gamma)}$$

$$Q(\hat{\beta}, \hat{p}_\Gamma, \hat{q}_\Gamma) = \mathrm{Re}\left\langle N\hat{p}_\Gamma + D^*(\mathrm{i}k\hat{\beta}\hat{p}_\Gamma) + \mathrm{i}k\hat{\beta}\, D\hat{p}_\Gamma + \mathrm{i}k\hat{\beta}\, S(\mathrm{i}k\hat{\beta}\hat{p}_\Gamma) - h_1^{\mathrm{in}}(\hat{\beta}), \hat{q}_\Gamma \right\rangle$$

Adjoint state equation $\dfrac{\partial \mathcal{L}}{\partial p_\Gamma}(\beta, p_\Gamma, q_\Gamma) = 0$ → <u>**dual BEM problem**</u>

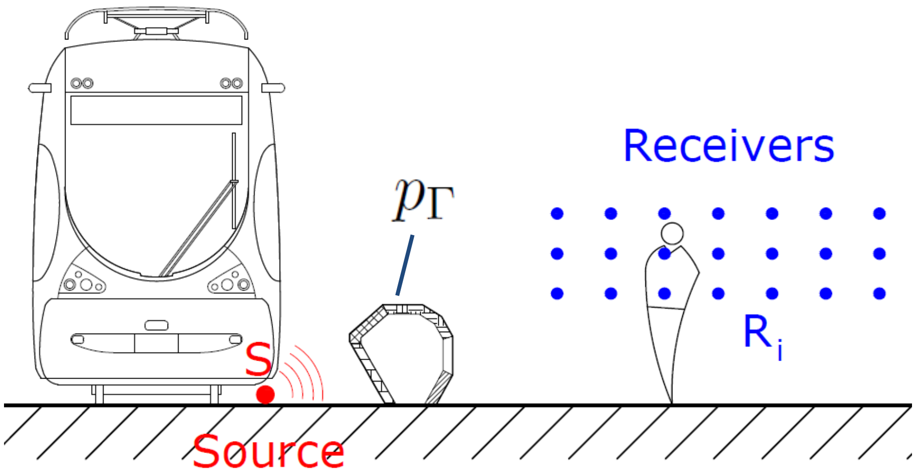
$$\boxed{N q_\Gamma + D^*(\mathrm{i}k\beta q_\Gamma) + \mathrm{i}k\beta\, D q_\Gamma + \mathrm{i}k\beta\, S(\mathrm{i}k\beta q_\Gamma) = h_2^{\mathrm{in}}(\beta, p_\Gamma)}$$

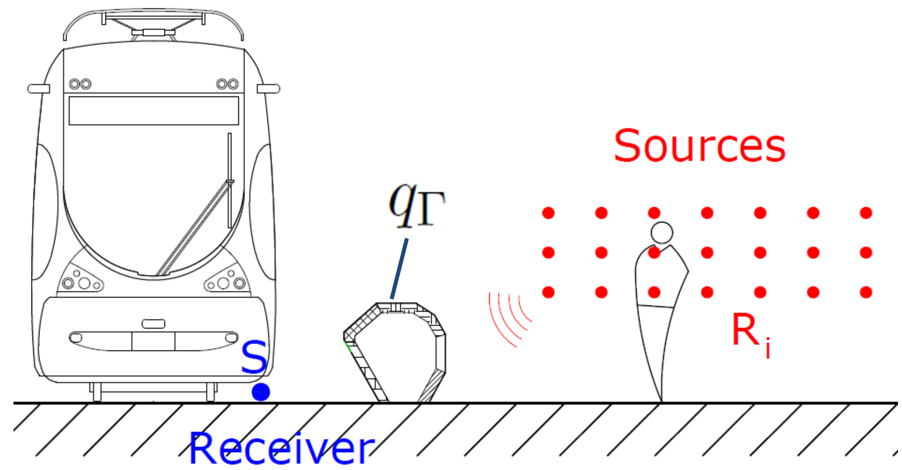
$$h_2^{\mathrm{in}}(\beta, p_\Gamma) = -\sum_i \frac{p(R_i)^*}{P}\left(\frac{\partial G}{\partial n_2}(R_i, .) + ik\beta\, G(R_i, .)\right)$$

Total gradient $\dfrac{\mathrm{d}P}{\mathrm{d}\beta}(\beta, p_\Gamma) = \dfrac{\partial \mathcal{L}}{\partial \beta}(\beta, p_\Gamma, q_\Gamma)$ → **Explicit function**

<u>**Source:**</u> G. Allaire, "Conception optimale des structures" (Optimal design of structures), Springer (2007)

The adjoint state is in fact the "state" of a dual scattering problem and can be solved by the BEM as well





$$p^{\text{in}} = G(S,.)$$

$$\begin{cases} -(\nabla^2 + k^2)\, p^{\text{sc}} = 0 \quad \text{in } \Omega^e \\ \dfrac{\partial p^{\text{sc}}}{\partial n} + ik\,\beta\, p^{\text{sc}} = -\dfrac{\partial p^{\text{in}}}{\partial n} - ik\,\beta\, p^{\text{in}} \stackrel{\text{def}}{=} h_1^{\text{in}}(\beta) \quad \text{on } \Gamma \\ + \text{radiation condition} \end{cases}$$

$$\boxed{N p_\Gamma + D^*(\mathrm{i}k\beta p_\Gamma) + \mathrm{i}k\beta\, D p_\Gamma + \mathrm{i}k\beta\, S(\mathrm{i}k\beta p_\Gamma) = h_1^{\text{in}}(\beta)}$$

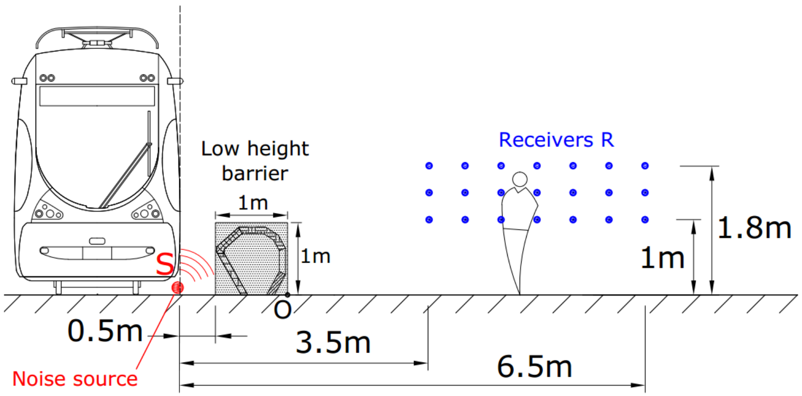
$$q^{\text{in}} = \sum_i \frac{p(R_i)^*}{P} G(R_i,.)$$

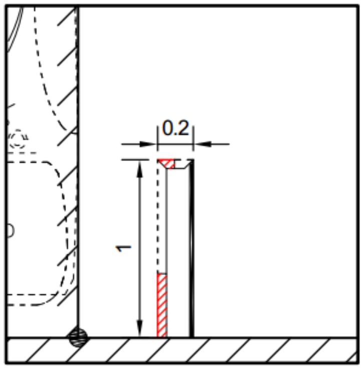
$$\begin{cases} -(\nabla^2 + k^2)\, q^{\text{sc}} = 0 \quad \text{in } \Omega^e \\ \dfrac{\partial q^{\text{sc}}}{\partial n} + ik\,\beta\, q^{\text{sc}} = -\dfrac{\partial q^{\text{in}}}{\partial n} - ik\,\beta\, q^{\text{in}} \stackrel{\text{def}}{=} h_2^{\text{in}}(\beta) \quad \text{on } \Gamma \\ + \text{radiation condition} \end{cases}$$

$$\boxed{N q_\Gamma + \mathrm{i}k\beta\, D q_\Gamma + D^*(\mathrm{i}k\beta q_\Gamma) + \mathrm{i}k\beta\, S(\mathrm{i}k\beta q_\Gamma) = h_2^{\text{in}}(\beta, p_\Gamma)}$$

→ **2 classical BEM integral equations: same operator, different RHS**

Results show good improvement of the barrier performance especially in the mid-frequency range

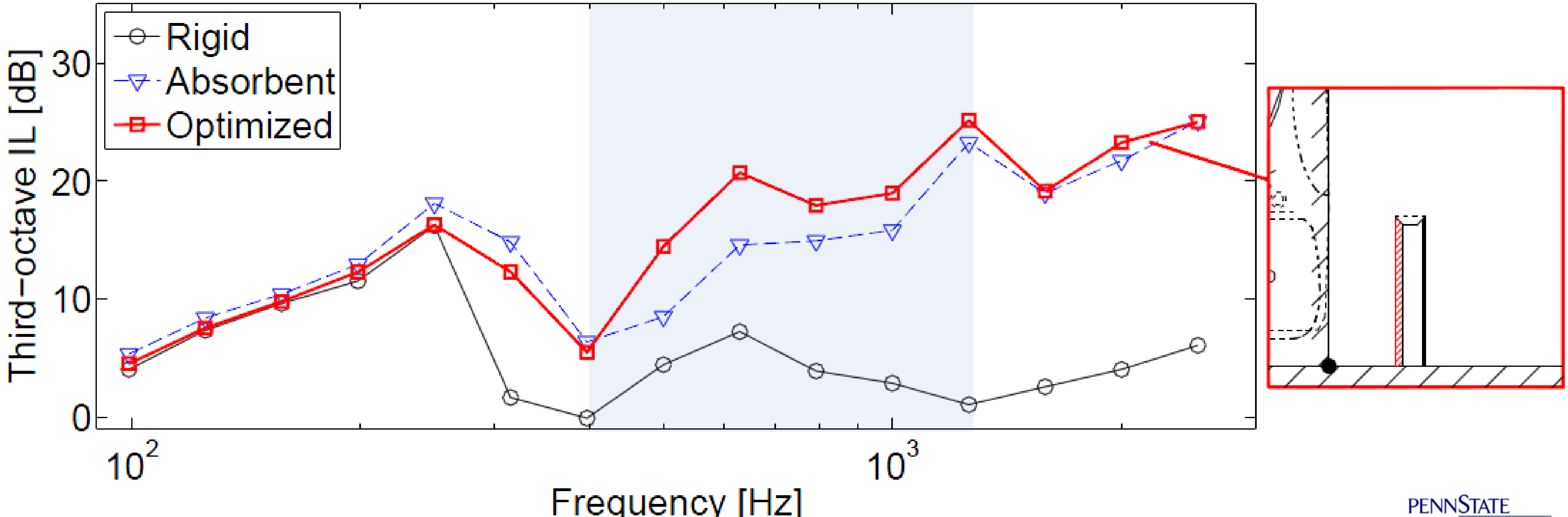




Thin wall

Rigid	3.7 dB(A)
Absorbent	14.3 dB(A)
Optimized	17.8 dB(A)

Optimization gain: 3.5 dB(A)



Absorbent: D&B layer – σ = 50 kPa s/m², d = 10 cm

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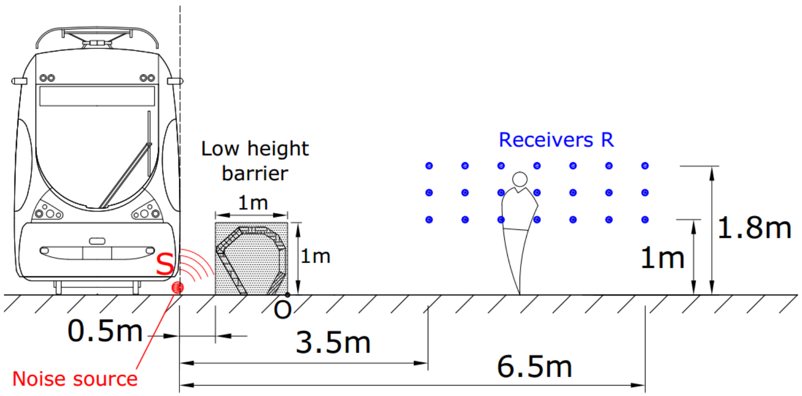
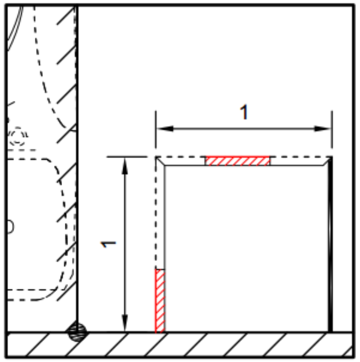
Results show good improvement of the barrier performance especially in the mid-frequency range

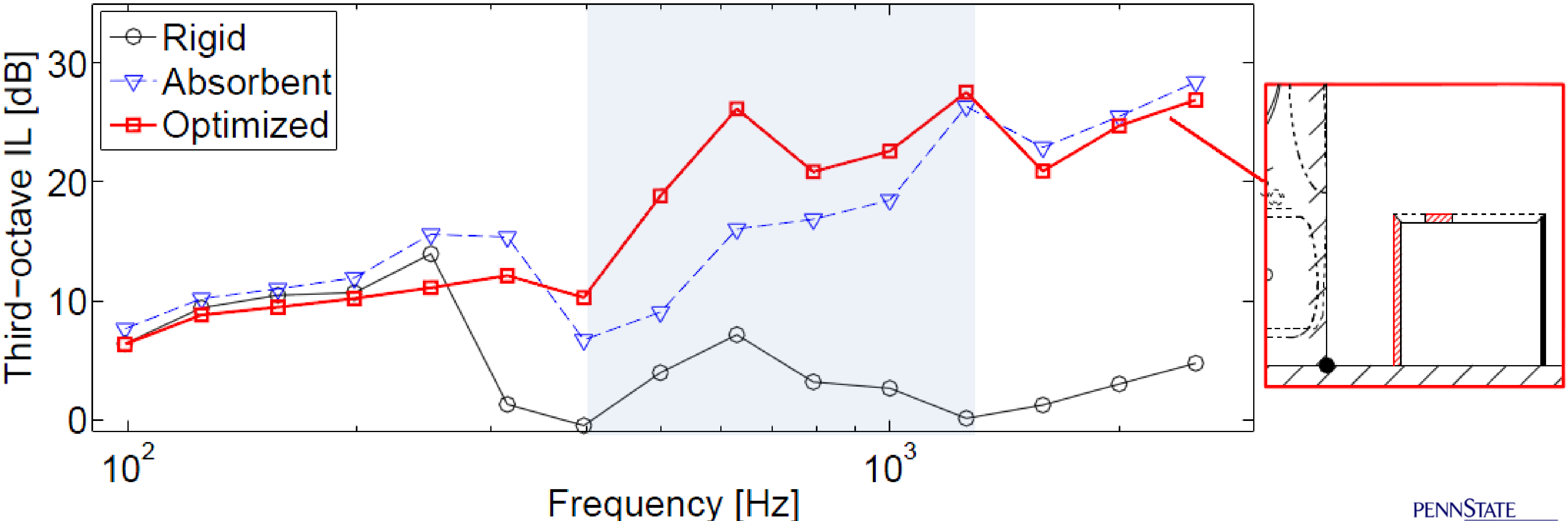




Square

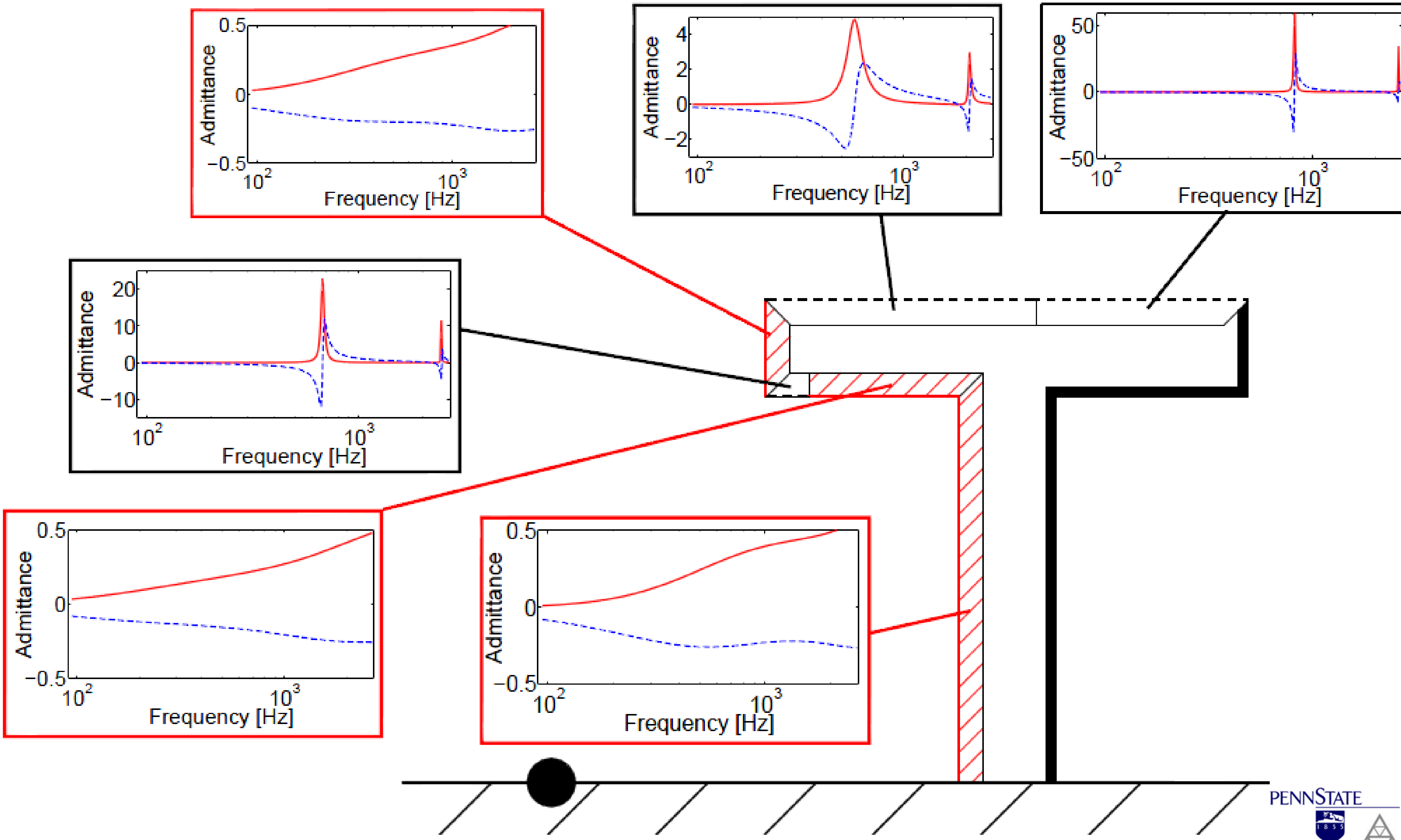
Rigid	3.1 dB(A)
Absorbent	15.7 dB(A)
Optimized	21.6 dB(A)

Optimization gain: 6 dB(A)

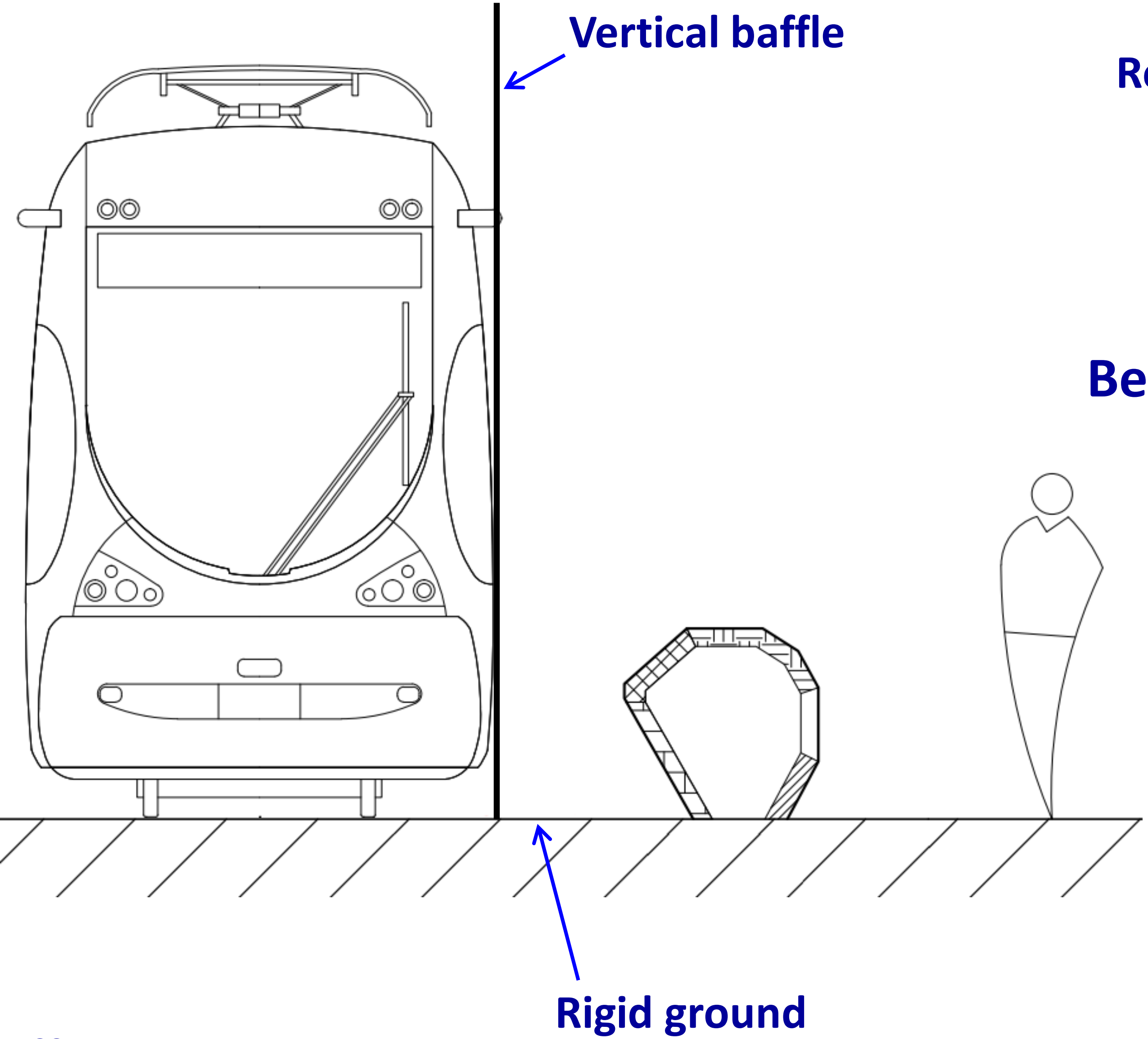


Absorbent: D&B layer – σ = 50 kPa s/m², d = 10 cm

Both surface treatments enhance attenuation in different frequency bands



One can also assess the accuracy of the approximations used in the optimization model



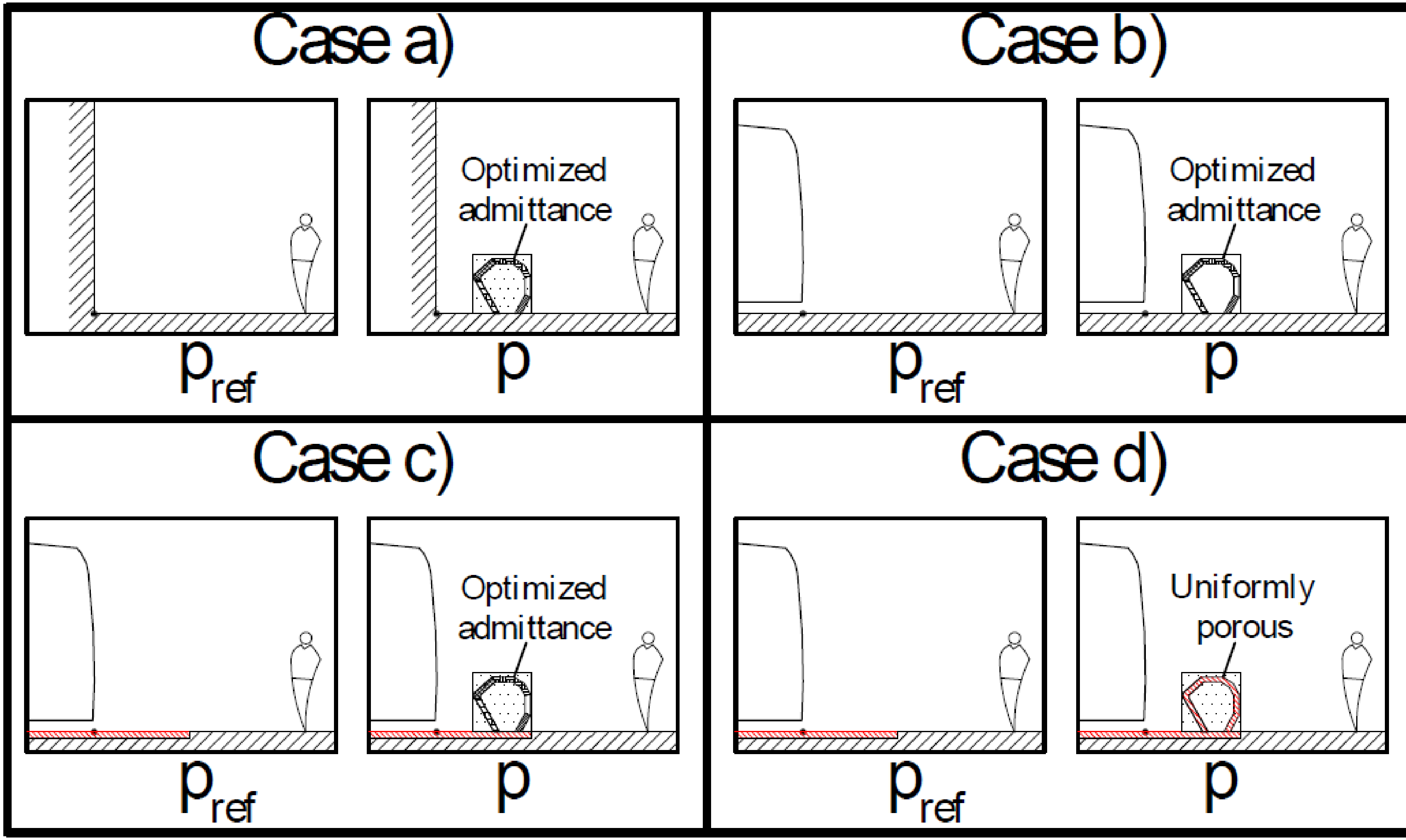
Absorbing ground ?

Realistic tram cross section ?

↓

Benefit of the optimized admittance ?

Different cases involving more realistic situations have been considered for comparison



Absorbing ground : Delany & Bazley layer – σ = 50 kPa s/m^2 , d = 10 cm

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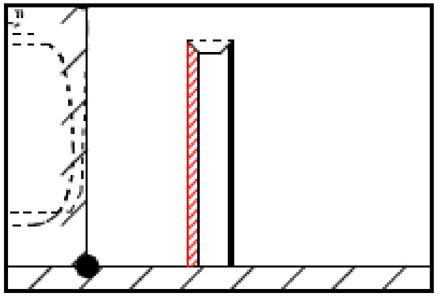
The benefit of the barrier is decreased when more realistic conditions are considered, but still significant

Thin wall



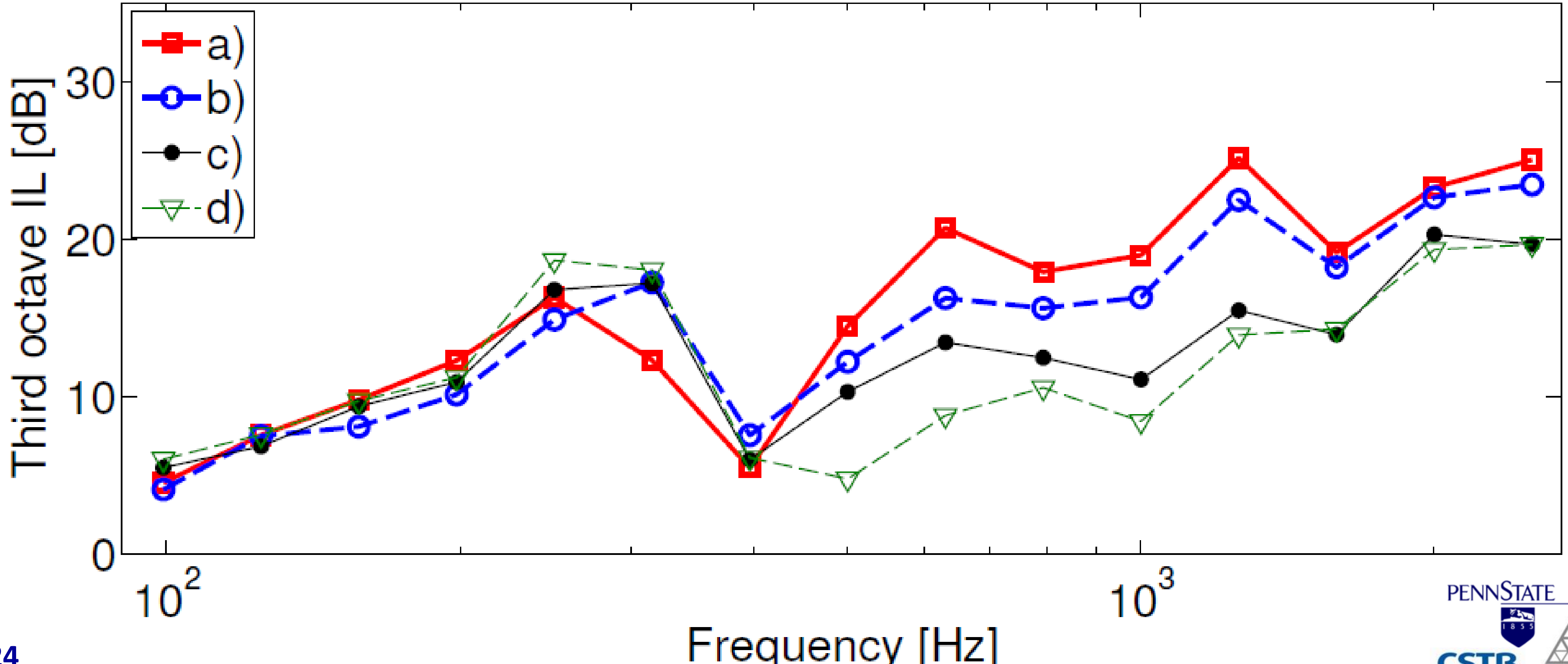
a) Vertical baffle, rigid ground, opt. β — IL_A = 17.8 dB(A)

b) Realistic tram, rigid ground, opt. β — IL_A = 15.8 dB(A)

c) Realistic tram, absorbing ground, opt. β — IL_A = 12.1 dB(A)

d) Realistic tram, absorbing ground, uniform absorbing β — IL_A = 9.0 dB(A)

Optimization gain : 3+ dB(A)



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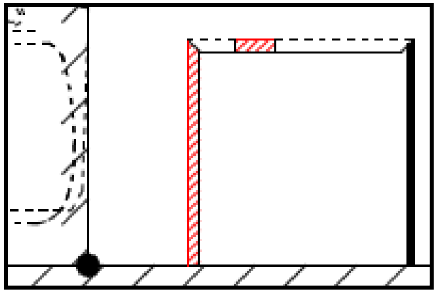
The benefit of the barrier is decreased when more realistic conditions are considered, but still significant

Square



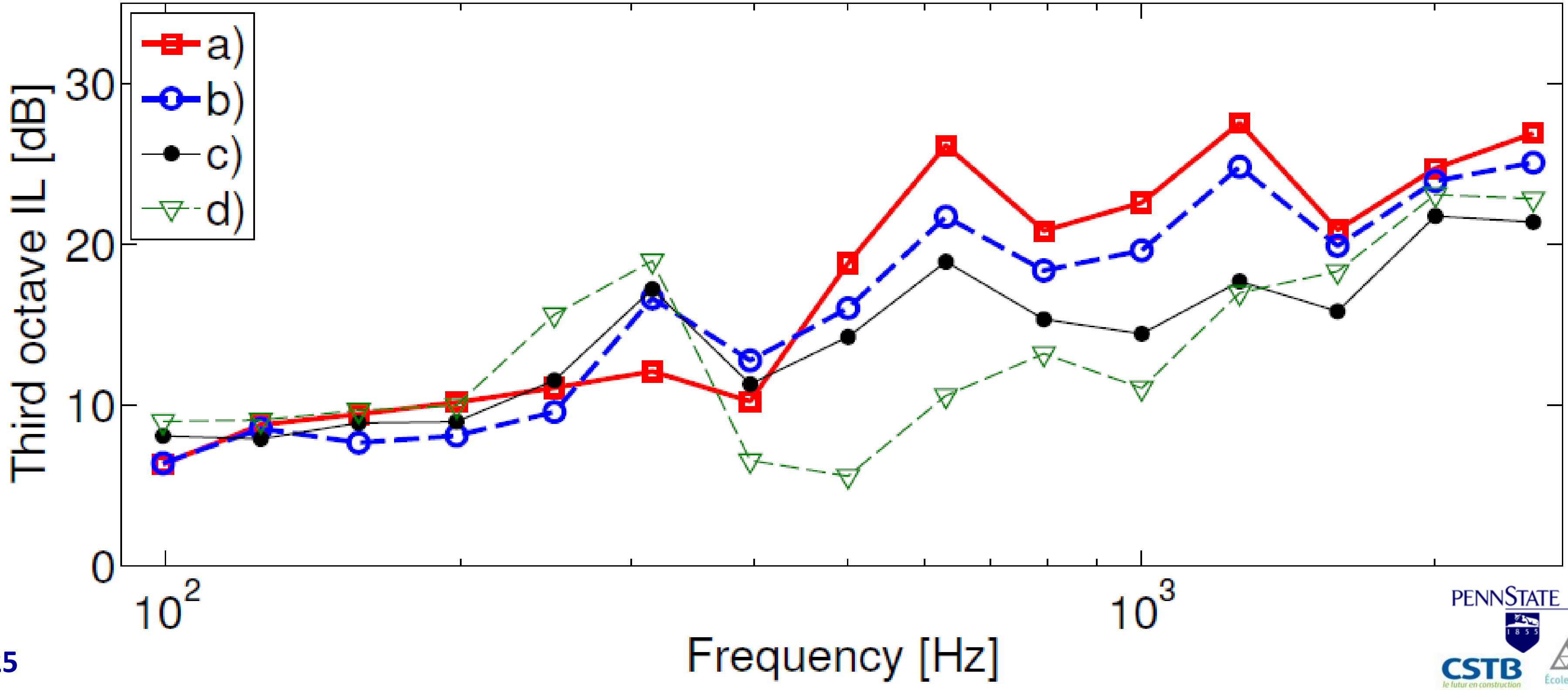
a) Vertical baffle, rigid ground, opt. β — IL_A = 21.6 dB(A)

b) <u>Realistic tram</u>, rigid ground, opt. β — IL_A = 19.3 dB(A)

c) Realistic tram, <u>absorbing ground</u>, opt. β — IL_A = 15.6 dB(A)

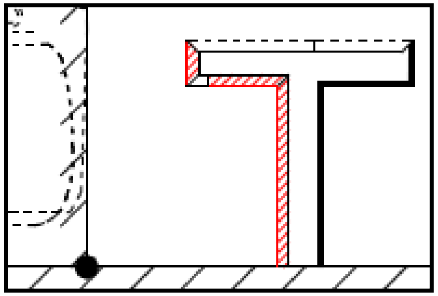
d) Realistic tram, absorbing ground, <u>uniform absorbing β</u> — IL_A = 10.9 dB(A)

Optimization gain : 4.5+ dB(A)



The benefit of the barrier is decreased when more realistic conditions are considered, but still significant

T-shape



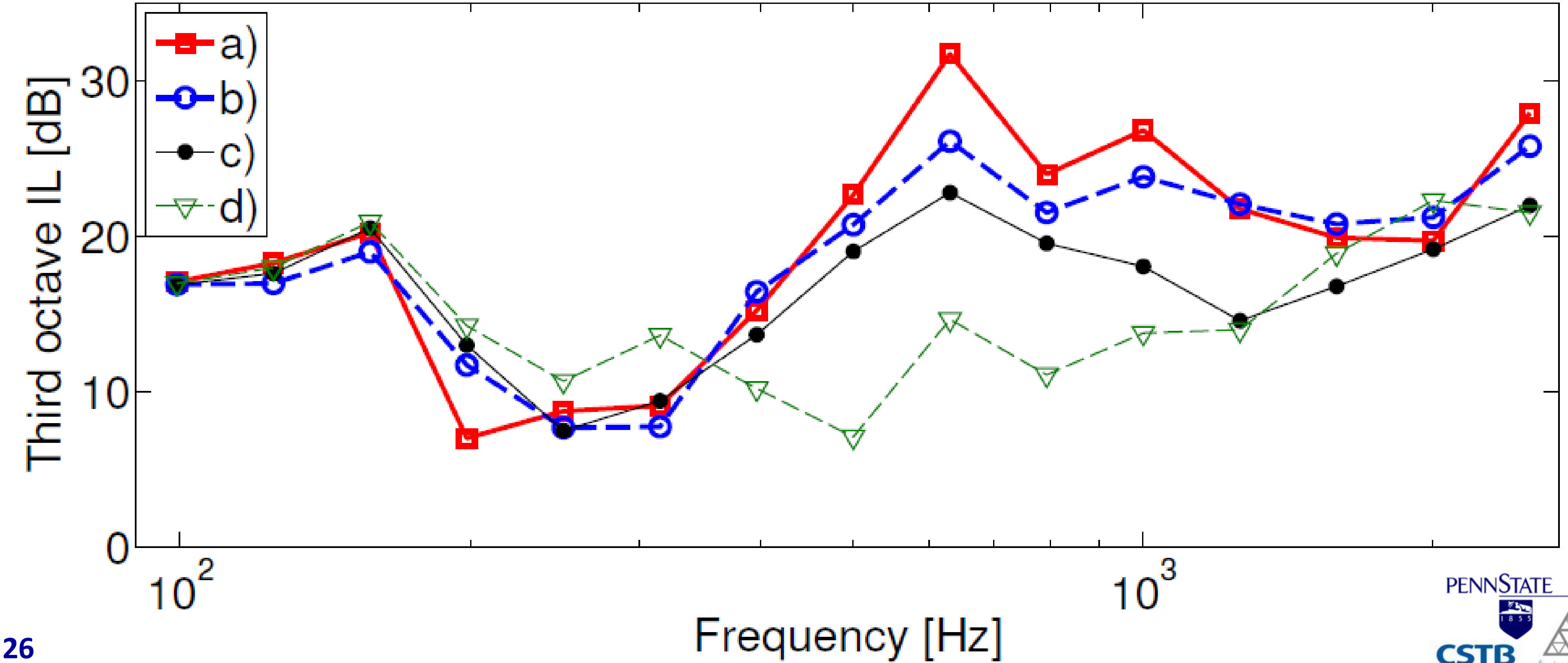
a) Vertical baffle, rigid ground, opt. β — IL_A = 23.1 dB(A)

b) Realistic tram, rigid ground, opt. β — IL_A = 22.2 dB(A)

c) Realistic tram, absorbing ground, opt. β — IL_A = 17.9 dB(A)

d) Realistic tram, absorbing ground, uniform absorbing β — IL_A = 12.4 dB(A)

Optimization gain : 5.5+ dB(A)



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